Some non-existence results on *m*-ovoids in finite classical polar spaces

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(joint work with Jan De Beule and Valentino Smaldore)

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Introduction

Polar spaces

Let $\mathrm{PG}(n, q)$ denote the n-dimensional projective space over the finite field $\mathrm{GF}(q)$.

Definition

A non-degenerate sesquilinear or non-singular quadratic form on the underlying $(n+1)$-dimensional vector space

Remark:

- Consists of the totally isotropic, respectively, totally singular subspaces.
- The subspaces of maximal dimension are called generators.
- If $r-1$ is the dimension of a generator, the the rank equals r.
- Induces a polarity $\perp$ of the ambient projective space.

Introduction

Some notations

Consider a *classical finite polar space* $\mathcal{P}_{r,e}$ in $\mathrm{PG}(n, q)$, where

polar space	notation	n	e
elliptic quadric	$\mathrm{Q}^-(2r+1, q)$	$2r+1$	2
hyperbolic quadric	$\mathrm{Q}^+(2r-1, q)$	$2r-1$	0
parabolic quadric	$\mathrm{Q}(2r, q)$	$2r$	1
symplectic space	$\mathrm{W}(2r-1, q)$	$2r-1$	1
Hermitian polar space	$\mathrm{H}(2r, q)$	$2r$	$3/2$
Hermitian polar space	$\mathrm{H}(2r-1, q)$	$2r-1$	$1/2$

Table: $\mathcal{P}_{r,e}$ polar space of rank $r \geq 1$

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Table: $\mathcal{P}_{r,e}$ polar space of rank $r \geq 1$

Note that $\mathcal{P}'_{r,e}$ stands for one of the polar spaces $\mathrm{W}(2r-1,q)$, $\mathrm{Q}^-(2r+1,q)$ or $\mathrm{H}(2r,q)$ (q square), i.e. $e \in \left\{1, \frac{3}{2}, 2\right\}$.

Introduction

m-ovoids

Definition

A set $\mathcal{O}$ of points of a polar space $\mathcal{P}_{r,e}$ is an *m-ovoid of* $\mathcal{P}_{r,e}$ if and only if every generator of $\mathcal{P}_{r,e}$ contains exactly m points of $\mathcal{O}$.

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Some results:

- $|\mathcal{O}| = m(q^{r+e-1}+1)$,
- In $\mathcal{P}'_{r,e}$, for every $p \in \mathrm{PG}(n,q)$

$$|p^{\perp} \cap \mathcal{O}| = \begin{cases} (m-1)(q^{r+e-2}+1)+1, & p \in \mathcal{O}, \\ m(q^{r+e-2}+1), & p \in \mathrm{PG}(n,q) \setminus \mathcal{O}\,. \end{cases}$$

m-ovoids and characteristic functions

Suppose that χ is the characteristic vector of $\mathcal{O}$. Then we can define the Boolean function

$$\mu : \mathrm{PG}(n, q) \to \{0, 1\},$$

such that for every subspace π it holds that $\mu(\pi) = \sum_{p \in \pi} \chi_p$.

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Consequence

In $\mathcal{P}'_{r,e}$ it holds for every point p in $\mathrm{PG}(n, q)$ that

$$\mu(p^\perp) + q^{r+e-2}\mu(p) = m(q^{r+e-2} + 1)\,.$$

Introduction

Generalization of the consequence

Lemma

Consider $\mathcal{P}'_{r,e}$, then for every j-dimensional space π in $\mathrm{PG}(n,q)$,

$$\mu(\pi^{\perp}) + q^{r+e-j-2}\mu(\pi) = m(q^{r+e-j-2}+1)\,.$$

Known results

Non-existence results

Theorem (Bamberg, Kelly, Law and Penttila, [1])

Consider an m-ovoid $\mathcal{O}$ in the polar space $\mathcal{P}'_{r,e}$. Then $m \geq b$, with b given in the table below.

$\mathcal{P}'_{r,e}$	b
$\mathrm{Q}^-(2r+1, q)$	$\frac{-3+\sqrt{9+4q^{r+1}}}{2(q-1)}$
$\mathrm{W}(2r-1, q)$	$\frac{-3+\sqrt{9+4q^{r}}}{2(q-1)}$
$\mathrm{H}(2r, q^2)$	$\frac{-3+\sqrt{9+4q^{2r+1}}}{2(q^2-1)}$

Known results

Non-existence results

Theorem (Bamberg, Kelly, Law and Penttila, [1])

Let $\mathcal{O}$ be a non-trivial m-ovoid of $H(4, q^2)$. If $q > 2$, then

$$m \geq \frac{1}{2}\frac{-3q - 3 + \sqrt{4q^5 - 4q^4 + 5q^2 - 2q + 1}}{q^2 - q - 2}.$$

While for $q = 2$, we have $m \geq 2$.

Analyzing m-ovoids

Main equation for points

Using a double counting argument based on

A. L. Gavrilyuk, K. Metsch, and F. Pavese.
A modular equality for m−ovoids of elliptic quadrics.
Bull. London Math. Soc., (10.1112/blms.12830), 2023.

Theorem

Suppose that μ is a m-ovoid in $\mathcal{P}'_{r,e}$ and let p_0 be an arbitrary point in $\mathcal{P}'_{r,e}$ such that $\mu(p_0) < m$. Then

$$m(q^{r+e-3}+1)(m(q^{r+e-1}+1)-\mu(p_0))+q^{r+e-2}\sum_{p\in p_0^\perp\setminus\{p_0\}}\mu(p)^2$$

$$=m(q^{r+e-2}+1)^2(m-\mu(p_0))+q^{r+e-3}\sum_{p\in\mathcal{P}'_{r,e}\setminus\{p_0\}}\mu(p)\mu(\langle p_0,p\rangle)$$

First improvements

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First improvements

Lemma (De Beule, JM and Smaldore)

Let $\mathcal{O}$ be a non-trivial m-ovoid in $\mathcal{P}'_{r,e}$, with weight function μ.

- *If $p_0 \in \mathcal{P}'_{r,e}$, then* $\sum_{p \in p_0^\perp \setminus \{p_0\}} \mu(p)^2 = (m - \mu(p_0))(q^{r+e-2} + 1)$,

Proof.

$$\mu(p_0^\perp \setminus \{p_0\}) = \mu(p_0^\perp) - \mu(p_0) = m(q^{r+e-2} + 1) - q^{r+e-2}\mu(p_0) - \mu(p_0)$$
$$= (m - \mu(p_0))(q^{r+e-2} + 1)\,.$$

□

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Suppose that μ is a m-ovoid in $\mathcal{P}'_{r,e}$ and let p_0 be an arbitrary point in $\mathcal{P}'_{r,e}$ such that $\mu(p_0) < m$. Then

$$m(q^{r+e-3}+1)(m(q^{r+e-1}+1)-\mu(p_0))+q^{r+e-2}(m-\mu(p_0))(q^{r+e-2}+1)$$
$$=m(q^{r+e-2}+1)^2(m-\mu(p_0))+q^{r+e-3}\sum_{p\in\mathcal{P}'_{r,e}\setminus\{p_0\}}\mu(p)\mu(\langle p_0,p\rangle)$$

First improvements

Lemma (De Beule, JM and Smaldore)

Let $\mathcal{O}$ be a non-trivial m-ovoid in $\mathcal{P}'_{r,e}$, with weight function μ.

- *If $p_0 \in \mathcal{O}$ then*

$$\sum_{p \in \mathcal{P}'_{r,e} \setminus \{p_0\}} \mu(p)\mu(\langle p_0, p \rangle) \geq 2(m(q^{r+e-1}+1)-1)\,.$$

Proof.

Let $p \in \mathcal{O} \setminus \{p_0\}$. Then $\mu(\langle p, p_0 \rangle) \geq \mu(p) + \mu(p_0) = 2$

□

First improvements

using the previous equation, we obtain the following inequality

$$(q-1)^2m^2 + 3(q-1)m - q^{r+e-1} - q - 2 \geq 0.$$

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Lets solve this!

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What about $\mu(p_0) < m$?

Only a problem when $m = 1$, but this can be excluded by previous results.

First improvements

First conclusion

Theorem (de Beule, JM and Smaldore)

Consider an m-ovoid $\mathcal{O}$ in the polar space $\mathcal{P}'_{r,e}$. Then $m \geq b$, with b given in the table below.

$\mathcal{P}'_{r,e}$	b
$\mathrm{Q}^-(2r+1,q)$	$\dfrac{-3+\sqrt{9+4(q^{r+1}+q-2)}}{2(q-1)}$
$\mathrm{W}(2r-1,q), r>2$	$\dfrac{-3+\sqrt{9+4(q^{r}+q-2)}}{2(q-1)}$
$\mathrm{H}(2r,q^2)$	$\dfrac{-3+\sqrt{9+4(q^{2r+1}+q^2-2)}}{2(q^2-1)}$

Improvement for $\mathrm{H}(4, q^2)$

Lemma (De Beule, JM and Smaldore)

Let $\mathcal{O}$ be a non-trivial m-ovoid in $\mathrm{H}(4, q^2)$ with weight function μ. Fix a point $p_0 \in \mathrm{H}(4, q^2) \cap \mathcal{O}$, then

$$\sum_{p \in H(4,q^2) \setminus \{p_0\}} \mu(p)\mu(\langle p_0, p \rangle) \geq m(m-1)(q^3+1) + 2(mq^3(q^2-1)+q^3)\,.$$

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Theorem (Bamberg, Kelly, Law and Penttila, [1])

Let $\mathcal{O}$ be an m-ovoid of $\mathrm{H}(4, q^2)$, for $q > 2$, then

$$m \geq \frac{-3q - 3 + \sqrt{4q^5 - 4q^4 + 5q^2 - 2q + 1}}{2(q^2 - q - 2)} .$$

Main results

Main equation

Theorem (De Beule, JM and Smaldore)

Suppose that μ is an m-ovoid in $\mathcal{P}'_{r,e}$ and let π be an arbitrary j-dimensional subspace, $0 \leq j \leq r-1$, with $\mu(\pi^\perp \setminus \pi) \neq 0$, then

$$m(q^{r+e-j-3}+1)(m(q^{r+e-1}+1)-\mu(\pi))+q^{r+e-2}\sum_{p\in\pi^\perp\setminus\pi}\mu(p)^2=$$

$$m(q^{r+e-2}+1)(m-\mu(\pi))(q^{r+e-j-2}+1)+q^{r+e-j-3}\sum_{p\in\mathcal{P}'_{r,e}\setminus\pi}\mu(p)\mu(\langle p,\pi\rangle)$$

$$+\sum_{s\notin\pi^\perp}\mu(s^\perp\cap\pi)\,.$$

Main results

Main equation. Choose $j = r - 2$

Theorem (De Beule, JM and Smaldore)

Suppose that μ is an m-ovoid in $\mathcal{P}'_{r,e}$ and let π be an arbitrary $(r-2)$-dimensional subspace, with $\mu(\pi^\perp \setminus \pi) \neq 0$, then

$$m(q^{e-1}+1)(m(q^{r+e-1}+1)-\mu(\pi))+q^{r+e-2}\sum_{p\in\pi^\perp\setminus\pi}\mu(p)^2=$$
$$m(q^{r+e-2}+1)(m-\mu(\pi))(q^e+1)+q^{e-1}\sum_{p\in\mathcal{P}'_{r,e}\setminus\pi}\mu(p)\mu(\langle p,\pi\rangle)$$
$$+\sum_{s\notin\pi^\perp}\mu(s^\perp\cap\pi)\,.$$

Main results

Lemma

If π is an $(r-2)$-space contained in $\mathcal{P}'_{r,e}$, then

$$\sum_{p\in\mathcal{P}'_{r,e}\setminus\pi}\mu(p)\mu(\langle p,\pi\rangle)\geq m(q^e+1)(m-\mu(\pi))+(1+\mu(\pi))(mq^e(q^{r-1}-1)+\mu(\pi)(q^e+1)).$$

Proof.

To prove this one needs to split the sum. Consider $\tau := \langle \pi, p\rangle$.

- If $p \in \pi^{\perp}$, then τ is a generator containing m points.

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- If $p \in \pi^\perp$, then τ is a generator containing m points. $\Rightarrow (q^e+1)(m-\mu(\pi))$ options.
- Otherwise $\mu(\langle \pi, p\rangle) \geq \mu(\pi) + \mu(p)$.

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- If $p \in \pi^\perp$, then τ is a generator containing m points.
  $\Rightarrow (q^e + 1)(m - \mu(\pi))$ options.
- Otherwise $\mu(\langle \pi, p\rangle) \geq \mu(\pi) + \mu(p)$.
  $\Rightarrow$ all other options.

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Theorem (De Beule, JM and Smaldore)

Suppose that μ is an m-ovoid in $\mathcal{P}'_{r,e}$ and let π be an arbitrary $(r-2)$-dimensional subspace, with $\mu(\pi^\perp \setminus \pi) \neq 0$, then

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$$m(q^{r+e-2}+1)(m-\mu(\pi))(q^e+1)+q^{e-1}\sum_{p\in\mathcal{P}'_{r,e}\setminus\pi}\mu(p)\mu(\langle p,\pi\rangle)$$
$$+\sum_{s\notin\pi^\perp}\mu(s^\perp\cap\pi)\,.$$

Main results

Lemma (De Beule, JM and Smaldore)

If π is an $(r-2)$-space contained in $\mathcal{P}'_{r,e}$, then

$$\sum_{s \notin \pi^\perp} \mu(s^\perp \cap \pi) = \mu(\pi) q^{r+2e-1} \frac{q^{r-2}-1}{q-1}.$$

Main results

Main equation. Choose $j = r - 2$

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Similar as before, we obtain that $\sum_{p\in\pi^\perp\setminus\pi}\mu(p)^2 = (m-\mu(\pi))(q^{r+e-j-2}+1)$.

Main results

Combining all results

Theorem (De Beule, JM and Smaldore)

Assume that $\mathcal{O}$ is an m-ovoid in $\mathcal{P}'_{r,e}$ and that π is an arbitrary $(r-2)$-space contained in $\mathcal{P}'_{r,e}$ such that $\mu(\pi^\perp \setminus \{\pi\}) \neq 0$, then

$$m^2(q^{r+e-1}-q^{r+e-2}-q^{2e-1}-q^e)$$
$$+m\big(\mu(\pi)(q^{r+e-2}+2q^{2e-1}+q^e)+q^{r+e-2}+q^{2e-1}\big)$$
$$-\mu(\pi)\left(q^{r+2e-2}+q^{r+e-2}+(1+\mu(\pi))(q^{2e-1}+q^{e-1})+q^{r+2e-1}\frac{q^{r-2}-1}{q-1}\right)\geq 0$$

Main results

Special case

Fill in the maximal $\mu(\pi)$ for good results.

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Fill in the maximal $\mu(\pi)$ for good results.

Lemma

Suppose that $\mathcal{O}$ is an m-ovoid in $\mathcal{P}'_{r,e}$, then there exist an $(r-2)$-space with at least $\min\{m, r-1\}$ points of $\mathcal{O}$.

Main results

Special case

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Suppose that $\mathcal{O}$ is an m-ovoid in $\mathcal{P}'_{r,e}$, then there exist an $(r-2)$-space with at least $\min\{m, r-1\}$ points of $\mathcal{O}$.

$\Rightarrow$ Use $\mu(\pi) = \min\{m, r-1\}$

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$\Rightarrow$ Use $\mu(\pi) = \min\{m, r-1\}$

- If $\mu(\pi) = m$, thus $m \leq r-1$.
  - If (a) $r \geq 4$, or, (b) $e \in \{1, \frac{3}{2}\}$ and $(r, q, e) \neq (3, 3, 1)$
    $\Rightarrow$ Contradiction with older results

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- If $\mu(\pi) = m$, thus $m \leq r-1$.
  - If (a) $r \geq 4$, or, (b) $e \in \{1, \frac{3}{2}\}$ and $(r, q, e) \neq (3, 3, 1)$
    $\Rightarrow$ Contradiction with older results
- So in these cases we can assume that $\mu(\pi) = r-1$.

Main results

conclusion

Theorem (De Beule, JM and Smaldore)

Let $q > 2$ and $r \geq 3$. Suppose that $\mathcal{O}$ is an m-ovoid in $\mathcal{P}'_{r,e}$, with (a) $r \geq 4$, or, (b) $e \in \{1, \frac{3}{2}\}$ and $(r, q, e) \neq (3, 3, 1)$. Then it holds that

$$m \geq \frac{-r(1+\frac{2}{q^{r-e}-1}) + \sqrt{r^2(1+\frac{2}{q^r-1})^2 + 4(q-2)(r-1)(q^{e+1}\frac{q^{r-2}-1}{q-1} + q^e + 1)}}{2(q-1)}.$$

This bound asymptotically converges to

$$m \geq \frac{-r + \sqrt{r^2 + 4(r-1)(q-2)q^{r+e-2}}}{2(q-1)}.$$

Main results

Some examples of the improvement

r	New bound	Old bound
4	$m \geq 5$	$m \geq 4$
5	$m \geq 10$	$m \geq 8$
6	$m \geq 20$	$m \geq 13$
7	$m \geq 39$	$m \geq 23$
100	$m \geq 2,53 \cdot 10^{24}$	$m \geq 3,59 \cdot 10^{23}$

Table: Bounds for m-ovoids of $W(2r-1,3)$

Main results

Some examples of the improvement

r	New bound	Old bound
4	$m \geq 8$	$m \geq 8$
5	$m \geq 18$	$m \geq 13$
6	$m \geq 36$	$m \geq 23$
7	$m \geq 69$	$m \geq 40$
100	$m \geq 4,37 \cdot 10^{24}$	$m \geq 6,22 \cdot 10^{23}$

Table: Bounds for m-ovoids of $Q^-(2r+1,3)$

Main results

Some examples of the improvement

r	New bound	Old bound
3	$m \geq 8$	$m \geq 6$
4	$m \geq 29$	$m \geq 18$
5	$m \geq 99$	$m \geq 53$
6	$m \geq 330$	$m \geq 158$
7	$m \geq 1085$	$m \geq 474$
100	$m \geq 1,04 \cdot 10^{48}$	$m \geq 1,12 \cdot 10^{47}$

Table: Bounds for m-ovoids of $H(2r, 3^2)$

References

John Bamberg, Shane Kelly, Maska Law, and Tim Penttila.
Tight sets and m-ovoids of finite polar spaces.
J. Combin. Theory Ser. A, 114(7):1293–1314, 2007.

Jan De Beule, Jonathan Mannaert and Valentino Smaldore.
Some non-existence results on m-ovoids in classical polar spaces.
submitted, 10.48550/arXiv.2305.06285

A. L. Gavrilyuk, K. Metsch, and F. Pavese.
A modular equality for $m-$ovoids of elliptic quadrics.
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Thank you for your attention!

Are there any questions?

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