# Another generalization for measure of fault tolerance in hypercubes

Rijeka Conference on Combinatorial Objects and Their Applications- 3-7 July, 2023

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July 6, 2023

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# Introduction

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Any interconnection network can be modeled into a graph in which vertices and edges represent processors and communication links, respectively.

Hypercubes, Augmented cubes and Multidimensional torus are some examples of interconnection networks.

We mainly focus on Hypercubes.

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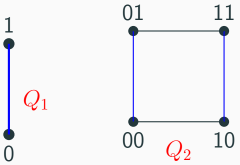
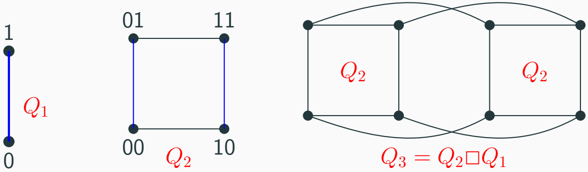
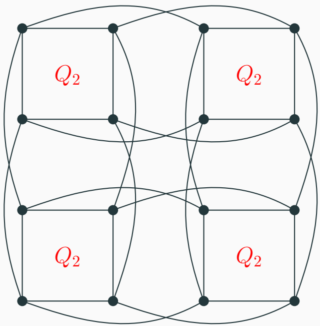


# $n$-dimensional hypercube $Q_n$



# Partition of hypercube $Q_4$ into $Q_2$

$Q_4 = Q_2 \square Q_2$



# Properties of hypercubes

| Properties | $Q_n$ |
|---|---|
| Order | $2^n$ |
| Size | $n2^{n-1}$ |
| Regularity | $n$ |
| Connectedness | $n$ |
| Cycles | Bipancyclic |
| Diameter | $n$ |

# Connectivity of graphs

- The connectivity of a graph measures the fault tolerance capacity and robustness of the network.
- For practical applications, a network having high fault tolerance capacity is usually preferred, and so, the underlying graph of the network is expected to have good connectivity.
- In traditional connectivities of a graph, isolation of a vertex from other vertices is allowed. However, in a practical situation, the failure of all the links incident to a node in an interconnection network is highly unlikely.
- To overcome this limitation, Harary [3] (1983) defined the conditional connectivity of a graph. This connectivity requires the components of the disconnected graph to satisfy certain properties.

# 1. Conditional connectivity

A conditional $h$-vertex cut of a graph $G$ is a set $F$ of vertices of $G$ such that the graph $G - F$ is disconnected, and each component of it has a minimum degree at least $h$. The conditional $h$-vertex connectivity of $G$, denoted by $\kappa^h(G)$, is the minimum cardinality of a conditional $h$-vertex cut of $G$.

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Oh et al. [6] and Wu et al.[9] independently determined the conditional $h$-connectivity of hypercube:

Theorem: For any $h$ in $0 \leq h \leq n-2$, $\kappa^h(Q_n) = 2^h(n-h)$.

# 2. Component connectivity

The component connectivity is introduced by Sampathkumar [7] (1984).

An $r$-component cut of $G$ is a set of vertices whose deletion results in a graph with at least $r$ components. The $r$-component connectivity $c\kappa_r(G)$ of a graph $G$ is the size of the smallest $r$-component cut of $G$.

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Result is due to Hsu et al. [4]

For $n \geq 2$, and $1 \leq r \leq n$, the $(r+1)$-component connectivity of hypercube is $rn - \dfrac{r(r+1)}{2} + 1$.

# 3. Structure connectivity

A set $F$ of connected subgraphs of $G$ is a subgraph-cut of $G$ if $G - V(F)$ is a disconnected or trivial graph. Let $H$ be a connected subgraph of $G$. Then $F$ is an $H$-structure-cut if $F$ is a subgraph-cut, and every element in $F$ is isomorphic to $H$. The $H$-structure-connectivity of $G$, denoted by $\kappa(G; H)$, to be the minimum cardinality of all $H$-structure-cuts of $G$.

# Previous results on structure connectivity of hypercubes

- By Lin et al.(2016), for $n \geq 3$, then $\kappa_s(Q_n, K_{1,1}) = n - 1$, and $\kappa_s(Q_n, K_{1,2}) = \dfrac{\lceil n \rceil}{2}$.
- Sabir and Meng (2018), $H \in \{K_{1,1}, K_{1,2}, K_{1,3}, C_4\}$
- Mane (2018), $H \in \{Q_m \subseteq Q_n \colon\ 1 \leq m \leq n - 1\}$.

# Combined connectivity

We introduce a type of connectivity which combines conditional [3], component [1, 7] and structure [5] connectivities.

Let $G$ be a connected graph and $r \geq 2$, $h \geq 0$ be integers. Let $S$ be a set of connected subgraphs of $G$ such that every member of $S$ is isomorphic to a connected subgraph $H$ of $G$. Then $S$ is called an $h$-conditional $r$-component $H$-structure cut of $G$, if there are at least $r$ connected components in $G - V(S)$ and each component has minimum degree at least $h$. The $h$-conditional $r$-component $H$-structure connectivity of $G$ is the minimum $|S|$ overall $h$-conditional $r$-component $H$-structure cut of $G$.

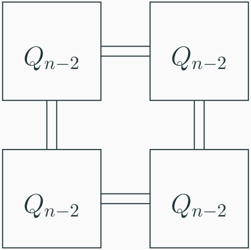
# Our result for hypercube

Let $m, n$ and $r$ be integers with $2 \leq r < m < n$.

Then $c\kappa_{r+1,m}(Q_n, Q_m)$ is

$$r(n-m) - \frac{r(r+1)}{2} + 1 \text{ for } 1 \leq r \leq n-m.$$

# Upper bound on the result

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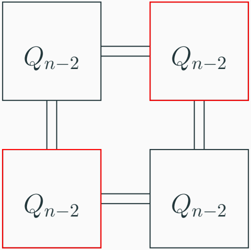


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# Thank You!