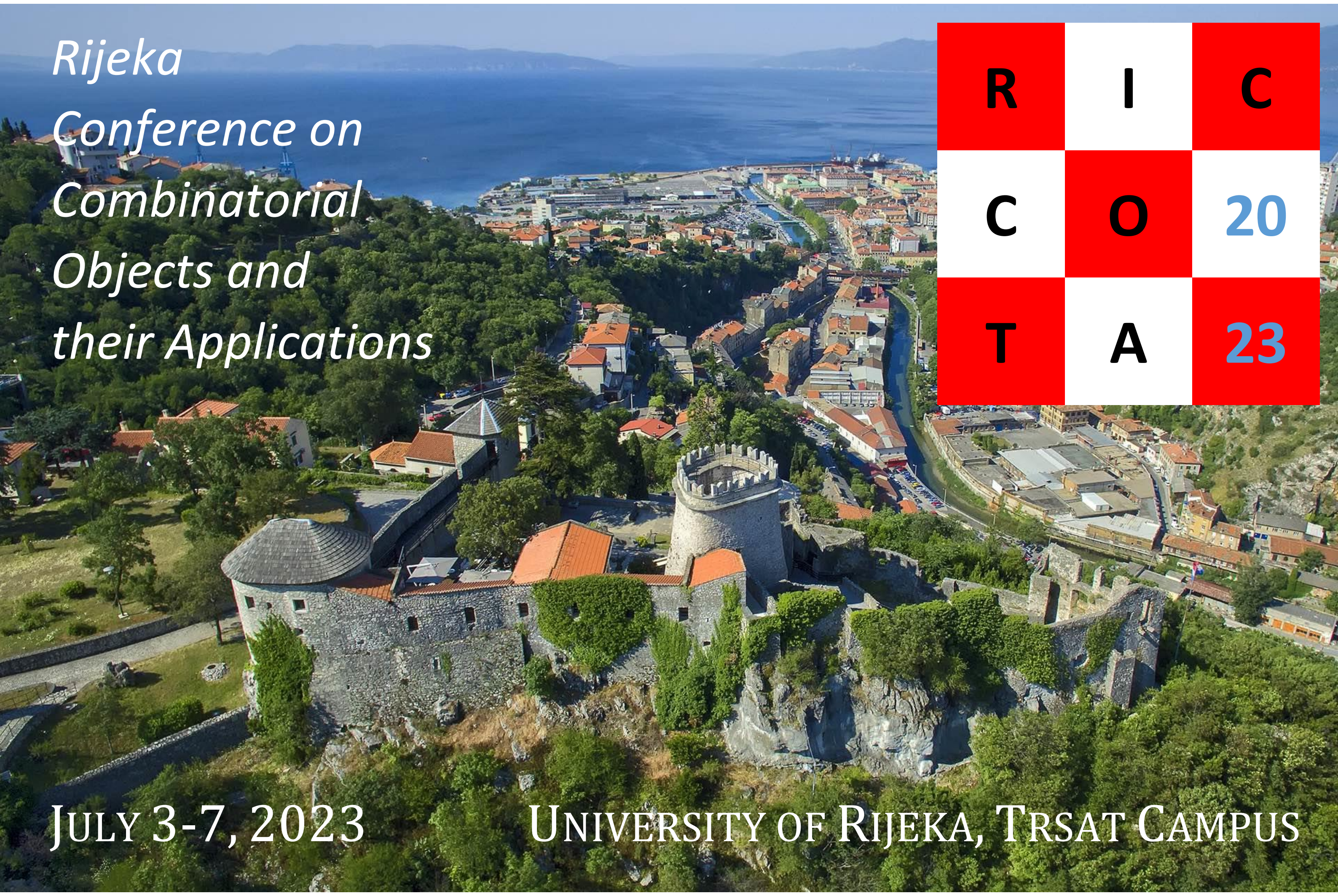

Rijeka Conference on Combinatorial Objects and their Applications

RICCOTA 2023

JULY 3-7, 2023

UNIVERSITY OF RIJEKA, TRSAT CAMPUS

Conference booklet

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UNIVERSITAS STUDIORUM FLUMINENSIS · UNIVERSITY OF RIJEKA

50 YEARS OF THE OPENNESS UNIVERSITY OF RIJEKA

UNIVERSITY OF RIJEKA FACULTY OF MATHEMATICS

General information

Goal and topics

The goal of the conference is to bring together researchers interested in design theory, finite geometry, graph theory, algebraic combinatorics and their applications to communication and cryptography, especially to codes (error-correcting codes, quantum codes, network codes, etc.). The conference will give the opportunity to research faculty and young researchers in discrete mathematics to learn about the latest developments in combinatorics and its contemporary applications in communication theory and information security, and to explore new directions for their future work.

Invited speakers

Aida Abiad, *Eindhoven University of Technology*
Ronan Egan, *Dublin City University*
Hadi Kharaghani, *University of Lethbridge*
Anamari Nakić, *University of Zagreb*
Francesco Pavese, *Polytechnic University of Bari*
Cheryl Praeger, *University of Western Australia*
Patrick Solé, *Aix-Marseille University*
Leo Storme, *Ghent University*
Vladimir Tonchev, *Michigan Technological University*

Organising Committee

Andrea Švob, *University of Rijeka* (chair)

Dean Crnković, *University of Rijeka*
Maarten De Boeck, *University of Memphis*
Daniel Hawtin, *University of Rijeka*
Vedrana Mikulić Crnković, *University of Rijeka*

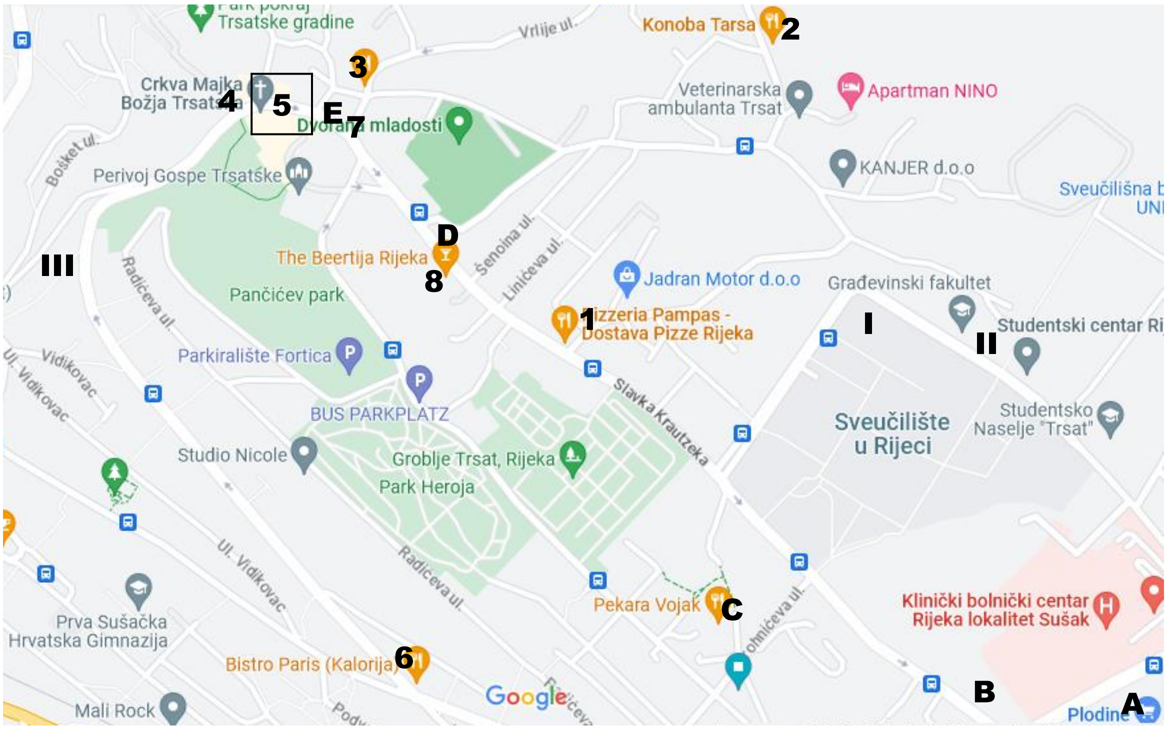
Room information

The conference takes place in the building of the Faculty of Mathematics on the Trsat campus of the University of Rijeka (where Sveučilišna Avenija makes a 90 degree turn). Talks are in rooms O-027 (ground floor) and O-S31 (floor -1). Plenary talks are in O-027. Coffee breaks are served in front of room O-027, and also the registration desk is there.

Internet access and conference website

Throughout the campus eduroam is available. In the conference building also the UNIRI wifi network is available (password: Uniri2019!!).
Conference website: `www.riccota2023.math.uniri.hr`

Food and transport information



General

I Conference location

II Student restaurant (first floor): they serve breakfast, lunch and dinner at reasonable prices

III Stairs to downtown (539 steps): many food options in the center (ca. 40 minute walk)

Shops and bakeries

A Plodine: big supermarket

B Studenac: grocery store

C Vojak: bakery

D Piko: bakery and limited shopping

E Mlinar: bakery

Restaurants and bars

1. Pampas: pizzeria (as good as Italian, but don't tell the Italians)
2. Konoba Tarsa: traditional Croatian
3. Carraca (pizza, grill and more) and Konoba MB (grill and *marende*)
4. Trsatica **(conference dinner location)**
5. Trsat center (near church, ca 10 min walk): several restaurants and bars
6. Bistro Paris
7. Gelateria Trsat: for the sweet tooth
8. Beertija: although there are many bars on Trsat, this is the one with the highest probability of finding math faculty on the terrace.

The walk from the campus to the center of Rijeka via the stairs is quite nice, but taking a bus or taxi back is also possible. There are several buses that run between the Trsat campus and the center of Rijeka (lines 2,7,8 and KBC). Information about these can be found on Google maps or on `https://www.autotrolej.hr/en/routes/`. The easiest way to order a taxi is via the Cammeo app (`https://cammeo.hr/en/ride-with-cammeo`).

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Conference programme

	Monday July 3		Tuesday July 4	
	O-027	O-S31	O-027	O-S31
8.15	Registration			
9.00	Opening		**Solé**	
9.15	**Praeger**			
10.00			coffee break	
10.15	coffee break (conference photo)			
10.30			Armario	D'haeseleer
10.55	Montinaro	Zelenyuk	Barrera-Acevedo	Simoens
11.20	Chen	Pisanski	Krčadinac	Gashi
11.45	Stojadinović		Ó Catháin	Wang
14.30	**Pavese**		**Nakić**	
15.30	coffee break		coffee break	
16.00	Browne	Pike	Burgess	Zeijlemaker
16.25	Smaldore	Fernández	Galici	Maleki
16.50	Mannaert	Zubović	Merola	Monzillo
17.15	Arumugam	Podrug	Tabak	Penjić

	Wednesday July 5		Thursday July 6		Friday July 7
	O-027	O-S31	O-027	O-S31	O-027
9.00	**Kharaghani**		**Storme**		**Egan**
10.00	coffee break		coffee break		coffee break
10.30	Wanless	Ernst	Lia	Panario	Torres-Martín
10.55	Falcón	Razafimahatratra	Adriaensen	Alonso-González	Mostarac
11.20	Buratti	Kempner	De Winter	Martinjak	Ban
11.45	Weiß	Islam		Beck	Mravić
14.00	Excursion				
14.30			**Abiad**		**Tonchev**
15.30			coffee break		coffee break
16.00			Khramova	Moura	Rodrigues
16.25			Lansdown	Shinde	Zrinski
16.50			Gavrilyuk	Clarke	Šumberac
17.15			Bailey	Bašić	Grbac
17.40					Closing
19.30			Conference dinner		

Monday July 3

8:15-9:00 Registration

9:00-9:15 Opening

9:15-10:10 **Cheryl Praeger: Novel constructions of normal covers of the complete bipartite graphs $K_{2^n,2^n}$**

10:15-10:55 Coffee break (conference photo)

ROOM O-027

10:55-11:15 Alessandro Montinaro: On flag-transitive symmetric 2-designs arising from Cameron-Praeger construction

11:20-11:40 Lei Chen: Vertex-primitive s-arc-transitive digraphs of almost simple groups

11:45-12:05 Tanja Stojadinović: The number of Hamiltonian paths in a digraph

ROOM O-S31

10:55-11:15 Yuliya Zelenyuk: Counting Symmetric Bracelets

11:20-11:40 Tomaž Pisanski: A Strategy for Generating Polycyclic Configurations

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14:30-15:25 **Francesco Pavese: On r-general sets in finite projective spaces**

15:30-16:00 Coffee break

ROOM O-027

16:00-16:20 Patrick Browne: Segre's theorem on ovals in Desarguesian projective planes

16:25-16:45 Valentino Smaldore: On regular systems of finite classical polar spaces

16:50-17:10 Jonathan Mannaert: Some non-existence results on m-ovoids in finite classical polar spaces

17:15-17:35 Vishnuram Arumugam: The Suzuki and Ree groups cannot act primitively on the points of a finite generalised quadrangle

ROOM O-S31

16:00-16:20 David Pike: Colourings of Path Systems

16:25-16:45 Blas Fernández: On the trivial T-module of a graph

16:50-17:10 Matea Zubović: Constructions of directed regular graphs from groups

17:15-17:35 Luka Podrug: Beyond Fibonacci cubes and Pell graphs

Novel constructions of normal covers of the complete bipartite graphs $\mathbf{K}_{2^n,2^n}$.

Cheryl E Praeger

University of Western Australia - Department of Maths & Stats

(Joint work with Dan Hawtin and Jin-Xin Zhou)

Abstract

It is thirty years since I introduced the concept of a normal quotient of a finite 2-arc-transitive graph, showing that each such graph is a normal cover of a 'basic' 2-arc-transitive graph. In the years since then a lot of progress has been made in identifying families of basic graphs. In particular, Cai Heng Li [1] identified all the basic 2-arc transitive graphs of prime power order, and posed the problem of characterising their 2-arc-transitive normal covers which also had prime power order: he stated that he was '*inclined to think that non-basic 2-arc-transitive graphs of prime power order would be rare and hard to construct*'. This was the motivation for the joint work which I will report on.

Our work focused on 2-arc-transitive normal covers of one of these basic families, namely the complete bipartite graphs $\mathbf{K}_{2^n,2^n}$. We first proved that such graphs are usually Cayley graphs, with all the exceptions based on a special family of groups called 'mixed dihedral groups'. We studied these mixed dihedral groups further and used them to build

- a new infinite family of 2-geodesic-transitive normal Cayley graphs which are neither distance-transitive nor 2-arc-transitive;
- a new infinite family of locally 2-arc-transitive semisymmetric graphs of 2-power order, (that is, provably not vertex-transitive);
- a 2-arc-transitive normal cover of $\mathbf{K}_{2^4,2^4}$ which is not a Cayley graph, and has order 2^{53}. We do not know if such graphs exist for $\mathbf{K}_{2^n,2^n}$ $(n > 4)$.

Bibliography

[1] C. H. Li. Finite s-arc transitive graphs of prime-power order. *Bull. London Math. Soc.*, **33**:129–137, 2001.

[2] D. R. Hawtin, C. E. Praeger and J.-X. Zhou.

(1) A characterisation of edge-affine 2-arc-transitive covers of $\mathbf{K}_{2^n,2^n}$. *ArXiv*:2211.16809, 2022.

(2) A family of 2-groups and an associated family of semisymmetric, locally 2-arc-transitive graphs. *ArXiv*:2303.00305, 2023.

(3) Using mixed dihedral groups to construct normal Cayley graphs, and a new bipartite 2-arc-transitive graph which is not a Cayley graph. *ArXiv*:2304.10633, 2023.

On flag-transitive symmetric 2-designs arising from Cameron-Praeger construction

Alessandro Montinaro

University of Salento

(Joint work with Cheryl E. Praeger)

Abstract

Based on a previous result of Sane [4], in 2016 Cameron and Praeger [3] provided a construction of a family of symmetric 2-designs with a specified point-partition. The authors also gave necessary and sufficient conditions for a 2-design $\mathcal{D}$ in the above mentioned family to possess a flag-transitive automorphism group G preserving the specified point-partition Σ and provided remarkable examples, one of them new. All the flag-transitive examples in [3] have the following property $\mathcal{R}$: *each block of imprimitivity* Δ *in* Σ *has the structure of an affine resolvable* 2*-design.* In this this talk, we show how to combine the results in [1, 2, 3] in order to obtain a classification of the pair $(\mathcal{D}, G)$ when $\mathcal{D}$ is in the Cameron-Praeger family and has the $\mathcal{R}$-property, and the group induced by G on Σ is not semilinear 1-dimensional.

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Vertex-primitive s-arc-transitive digraphs of almost simple groups

Lei Chen

University of Western Australia

Abstract

The property of s-arc-transitivity has been well studied for many years. Weiss proved that finite undirected graphs that are not cycles can be at most 7-arc-transitive. On the other hand, Praeger showed that for each s there are infinitely many finite s-arc-transitive digraphs that are not $(s+1)$-arc-transitive. However, G-vertex-primitive (G,s)-arc-transitive digraphs for large s seem rare. Thus we are interested in finding an upper bound for such s. In 2018, Giudici and Xia showed that it is sufficient to determine s when G is almost simple. We will show that $s \leq 1$ when G is almost simple with socle $Sz(2^{2n+1})$ or ${}^2G_2(3^{2n+1})$ for $n \geq 1$.

The number of Hamiltonian paths in a digraph

Tanja Stojadinović

University of Belgrade - Faculty of Mathematics

(Joint work with Vladimir Grujić and Marko Pešović)

Abstract

In a series of recent talks, Richard Stanley introduced a symmetric function U_X associated to a digraph X, see [3]. This symmetric function enumerates descent sets of permutations corresponding to a digraph. Stanley and Grinberg expand the function U_X in the power sum basis of the algebra of symmetric functions and relate the number of Hamiltonian paths in the digraph X with the alternating sum of the coefficients in this expansion. In fact, the function U_X is related to two classical results in graph theory. The first of them originates in 1933 paper of Laszlo Redei [1] and states that every finite tournament contains an odd number of Hamiltonian paths. The other one was found by Claude Berge [2] and it claims that the numbers of Hamiltonian paths in a digraph and its complement are of the same oddity.

We construct a structure of combinatorial Hopf algebra on digraphs for which the enumerator U_X is obtained from a universal morphism to quasisymmetric functions. In our construction, the number of Hamiltonian paths in the complement $\overline{X}$ of a digraph X is actually the value of the character $\zeta(X)$ of the newly constructed combinatorial Hopf algebra of digraphs.

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Counting Symmetric Bracelets

Yuliya Zelenyuk

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Abstract

An r-ary bracelet of length n is an equivalence class of r-colorings of vertices of a regular n-gon, taking all rotations and reflections as equivalent. A bracelet is symmetric if a corresponding coloring is invariant under some reflection. We show that the number of symmetric r-ary bracelets of length n is $\frac{1}{2}(r+1)r^{\frac{n}{2}}$ if n is even, and $r^{\frac{n+1}{2}}$ if n is odd [1, 2]. We also discuss further developments of this topic [3].

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A Strategy for Generating Polycyclic Configurations

Tomaž Pisanski

University of Primorska - FAMNIT

(Joint work with Leah Berman and Gábor Gévay)

Abstract

In 1950, the renowned geometer H. S. M. Coxeter [4] clearly established a connection between configurations and graphs. In 2009, another great geometer, B. Grünbaum in his monograph [5], developed the contemporary theory of configurations of points and lines. For the first time he made a clear distinction between combinatorial configurations as part of incidence geometry and geometric configurations of points and lines in the real projective plane. Moreover, in between he placed topological configurations, realized with points and pseudolines in the real projective plane as part of the class of pseudoline arrangements. In 2003, in the paper [3], the class of configurations investigated mainly by Grünbaum and his student Leah Berman was named *polycyclic configurations*, their combinatorial version admitting a convenient description by voltage graphs and cyclic covering graphs. The graph-theoretic approach towards configurations has been presented in [6]. Berman developed a series of techniques for constructing geometric polycyclic configurations, eg., see [1]. In a recent paper [2], we employed various tools to investigate a novel polycyclic (21_4) configuration; the existence of this configuration disproves a conjecture of Grünbaum. The purpose of this talk is to present the strategy that we used.

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[3] M. Boben and T. Pisanski. Polycylic configurations. *European J. Combin.* **24**: 431–457, 2003.

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On r-general sets in finite projective spaces

Francesco Pavese

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Abstract

Let $\mathrm{PG}(n,q)$ denote the n-dimensional projective space over the finite field with q elements. A *cap* is a set of points in $\mathrm{PG}(n,q)$ such that at most two of them are on a line, whereas a set of points in $\mathrm{PG}(n,q)$ with at most n on a hyperplane is known as an *arc*. These objects have been extensively studied due to their connections to coding theory. More generally an r*-general set* in $\mathrm{PG}(n,q)$ is a set $\mathcal{X}$ of points spanning the whole $\mathrm{PG}(n,q)$ such that any r of them are in general position. Hence a cap is a 3-general set and an arc is an $(n+1)$-general set. An r-general set of $\mathrm{PG}(n,q)$ is called *complete* if it is not contained in a larger r-general set of $\mathrm{PG}(n,q)$. The study of r-general sets is not only of geometrical interest, but arises from coding theory. Indeed, let $\mathcal{X}$ be a complete r-general set of $\mathrm{PG}(n,q)$ of size k, such that there is an $(r-1)$-dimensional projective subspace of $\mathrm{PG}(n,q)$ containing $r+1$ points of $\mathcal{X}$. By identifying the representatives of the points of $\mathcal{X}$ with the columns of a parity check matrix of a q-ary linear code, there corresponds a non-extendable linear $[k, k-n-1, r+1]_q$ code with covering radius $r-1$.

One of the main issue is to determine the spectrum of the sizes of complete r-general sets in $\mathrm{PG}(n,q)$ and in particular their maximal and minimal possible values. In this talk I will focus on the cases when r equals 3 or 4. In particular, I will discuss recent results regarding small complete 3-general sets in $\mathrm{PG}(4d+1,q)$ and large complete 4-general sets in $\mathrm{PG}(2d+1,3)$ and $\mathrm{PG}(n,4)$ whose sizes are close to their respective trivial lower and upper bounds.

Segre's theorem on ovals in Desarguesian projective planes

PATRICK BROWNE

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(Joint work with Steven T. Dougherty, Padraig Ó Catháin)

Abstract

Segre's theorem on ovals in projective spaces is an ingenious result from the mid-twentieth century which requires surprisingly little background to prove. In this brief talk we give a self contained proof of Segre's theorem. This is accessible to most yet showcases some minor improvements to Segre's proof that allow for results in shorter time and simpler computations than the original. See below for all needed references.

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On regular systems of finite classical polar spaces

VALENTINO SMALDORE

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(Joint work with A. Cossidente, G. Marino and F. Pavese)

Abstract

Let $\mathcal{P}_{d,e}$ be a finite classical polar space of rank d, where we choose $e \in \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ as follows:

$\mathcal{P}_{d,e}$	$Q^+(2d-1,q)$	$W(2d-1,q)$	$Q(2d,q)$	$Q^-(2d+1,q)$	$H(2d-1,q)$	$H(2d,q)$
e	0	1	1	2	$\frac{1}{2}$	$\frac{3}{2}$

Table 1: Classification of finite classical polar spaces $\mathcal{P}_{d,e}$.

An m-regular system with respect to $(k-1)$-dimensional projective spaces of $\mathcal{P}$, $1 \leq k \leq d-1$, is a set $\mathcal{R}$ of generators of $\mathcal{P}$ with the property that every $(k-1)$-dimensional projective space of $\mathcal{P}$ lies on exactly m generators of $\mathcal{R}$. In this talk regular systems of polar spaces are investigated, giving three different construction methods of regular systems w.r.t. points of various polar spaces: we find hemisystems of elliptic quadrics by partitioning generators of an elliptic quadric into generators of hyperbolic quadrics embedded in it; we give a construction of regular systems by means of a k-system, in hyperbolic quadrics and in the parabolic quadric $Q(6,3)$; we find regular systems of hyperbolic and elliptic quadrics arising from the field reduction map.

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Some non-existence results on m-ovoids in finite classical polar spaces

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(Joint work with Jan De Beule and Valentino smaldore)

Abstract

An m-ovoid in a finite classical polar space is a set $\mathcal{O}$ of points such that each generator meets $\mathcal{O}$ in exactly m points. These objects are quite rare and hence one of the main research questions is to find non-existence conditions on the parameter m. In [3] the authors prove a modular equality on the parameter m for m-ovoids in the elliptic quadric $Q^-(2r+1,q)$. Using similar techniques with some major adjustments, we are able to generalize their combinatorial arguments to $H(2r,q)$ and $W(2r+1,q)$. Using this approach, we can improve the lower bound on m for the hermitian polar space $H(2r,q)$, the symplectic polar space $W(2r+1,q)$ and the elliptic quadric $Q^-(2r+1,q)$ found in [1]. In [2], we show for $q>2$ and $r \geq 3$ (or $r>3$ in some very particular cases) that

$$m \geq \frac{-r(1+\frac{2}{q^{r-e-1}}) + \sqrt{r^2(1+\frac{2}{q^{r-1}})^2 + 4(q-2)(r-1)(q^{e+1}\frac{q^{r-2}-1}{q-1} + q^e + 1)}}{2(q-1)}.$$

This result is a direct improvement of all known bounds so far. More interesting is that previously known bounds can also be derived from the same result. Hence our approach unifies many known non-existence results. In this talk we will focus on the proof and techniques described in [2].

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The Suzuki and Ree groups cannot act primitively on the points of a finite generalised quadrangle

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Supervised by Michael Giudici and John Bamberg.

Abstract

Incidence geometry is the study of geometric structures involving a collection of points and lines along with a relation (called incidence) which tells us whether a point lies on a line. A generalised polygon is a type of point-line incidence structure that was introduced by Jacques Tits in 1959 to study the groups of Lie type as the symmetries of geometric objects. Since then, these objects have been studied extensively in the areas of group theory and finite geometry. The classification of these objects started from Weiss and Tits and many results about the existence (and non-existence) of generalised polygons under various symmetry conditions (point primitivity, flag transitivity and so on) since then. My aim is to show that the Suzuki and the Ree groups cannot act primitively on the points of a finite generalised quadrangle (which is a generalised 4-gon).

Colourings of Path Systems

David Pike

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(Joint work with Iren Darijani)

Abstract

By P_m we denote a path on m vertices. A P_m system of order n consists of a partition of the edge set of the complete graph K_n into subgraphs, each of which is isomorphic to P_m. A c-colouring of a path system is a function ϕ from the vertex set of K_n to a set C of c "colours" with the added restriction that each path in the path system must receive at least two colours. If a system can be c-coloured but not $(c-1)$-coloured, then we say that it is c-chromatic, and that its chromatic number is c. We prove that each any $c \geq 2$ and each $m \geq 3$ there exists a c-chromatic P_m system. In the case of $c \geq 3$ and $m = 4$ we also show that for each sufficiently large admissible order n there exists a c-chromatic P_4 system of order n. We also study unique colourings and show that there is a uniquely 2-chromatic P_4 system of order n for each $n \equiv 0$ or $1 \pmod 3$ such that $n \geq 109$.

On the trivial T-module of a graph

Blas Fernández
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(Joint work with Štefko Miklavič)

Abstract

Let Γ denote a finite, simple and connected graph. Fix a vertex x of Γ and let $T = T(x)$ denote the Terwilliger algebra of Γ with respect to x. In many instances, the algebra T can best be studied via its irreducible modules. In this talk, we will study the unique irreducible T-module with endpoint 0. It was already proved in [4] that this irreducible T-module is thin if Γ is distance-regular around x. The converse, however, is not true. Fiol and Garriga [3] later introduced the concept of *pseudo-distance-regularity* around vertex x, which is based on assigning weights to the vertices where the weights correspond to the entries of the (normalized) positive eigenvector. They showed that the unique irreducible T-module with endpoint 0 is thin if and only if Γ is pseudo-distance-regular around x (see also [2, Theorem 3.1]). To start our investigations, we give a purely combinatorial characterization of the property, that the unique irreducible T-module with endpoint 0 is thin. This characterization involves the number of walks of a certain shape between vertex x and vertices at some fixed distance from x. This is based on joint work with Štefko Miklavič [1].

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Constructions of directed regular graphs from groups

Matea Zubović

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(Joint work with Vedrana Mikulić Crnković)

Abstract

Regular directed graph Γ of degree k with n vertices is directed strongly regular graph, $DSRG(n, k, \lambda, \mu, t)$, if number of directed paths of length two from every vertex v to every vertex w is λ if there exists directed edge $v \to w$, t if $v = w$ and μ if there is no edge $v \to w$. Directed strongly regular graphs were introduced by Art Duval in 1988. One can construct 1-design by defining a basic block as union of G_α-orbits of transitive permutation group. Using that, we construct directed regular and strongly regular graphs from transitive groups.

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Beyond Fibonacci cubes and Pell graphs

Luka Podrug

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(Joint work with Tomislav Došlić)

Abstract

In his recent paper [1], Emanuele Munarini introduced a new family of graphs whose number of vertices corresponds to Pell numbers. His idea inspired us to investigate one possible generalization, by considering linear recurrences $s_n = a \cdot s_{n-1} + s_{n-2}$ for general values of the parameter a, going thus beyond the Fibonacci ($a = 1$) and Pell ($a = 2$) case. The resulting families of graphs turn out to exhibit many interesting properties. Some of them, both structural and enumerative, will be presented in this talk.

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Tuesday July 4

9:00-9:55 **Patrick Solé: A notion of bent sequences based on Hadamard matrices**

10:00-10:30 Coffee break

ROOM O-027

10:30-10:50 José Andrés Armario: Self-dual Butson bent sequences

10:55-11:15 Barrera-Acevedo: A Framework for Classifying Cocyclic Hadamard Matrices of order $8p$

11:20-11:40 Vedran Krčadinac: New constructions of higher dimensional Hadamard matrices and SBIBDs

11:45-12:05 Padraig Ó Catháin: The Hadamard maximal determinant problem

ROOM O-S31

10:30-10:50 Jozefien D'haeseleer: The chromatic number of some generalized Kneser graphs

10:55-11:15 Robin Simoens: Minimum weight of the code from intersecting lines in $\mathrm{PG}(3, q)$

11:20-11:40 Qëndrim R. Gashi: On a Problem Involving Strongly Orthogonal Roots

11:45-12:05 Zeying Wang: Some Results on Partial Difference Sets

14:30-15:25 **Anamari Nakić: On the additivity of 2-(v, k, λ) designs**

15:30-16:00 Coffee break

ROOM O-027

16:00-16:20 Andrea Burgess: Burning Steiner triple systems

16:25-16:45 Mario Galici: Extensions of Steiner Loops of Projective Type

16:50-17:10 Francesca Merola: Harmonious coloring of the incidence graph of a design

17:15-17:35 Kristijan Tabak: Dual incidences arising from a subsets of spaces

ROOM O-S31

16:00-16:20 Sjanne Zeijlemaker: On the diameter and zero forcing number of some graph classes in the Johnson, Grassmann and Hamming association scheme

16:25-16:45 Roghayeh Maleki: Distance-regular graphs with classical parameters which support a uniform structure: case $q \leq 1$ (Part 1)

16:50-17:10 Giusy Monzillo: Distance-regular graphs with classical parameters that support a uniform structure: case $q \geq 2$ (Part 2)

17:15-17:35 Safet Penjić: On (non)symmetric association schemes and associated family of graphs

A notion of bent sequences based on Hadamard matrices

Patrick Solé

Aix Marseille University

(Joint work with Wei Cheng, D. Crnković, Yaya Li, Denis Krotov, Minjia Shi)

A new notion of bent sequence related to Hadamard matrices was introduced recently, motivated by a security application (Solé et al, 2021). We study the self dual class in length at most 196. We use three competing methods of generation: Exhaustion, Linear Algebra and Groebner bases. Regular Hadamard matrices and Bush-type Hadamard matrices provide many examples. We conjecture that if v is an even perfect square, a self-dual bent sequence of length v always exist. We introduce the strong automorphism group of Hadamard matrices, which acts on their associated self-dual bent sequences. We give an efficient algorithm to compute that group. A generalization to complex Hadamard matrices is sketched out.

Self-dual Butson bent sequences

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(Joint work with Ronan Egan and Padraig Ó Catháin)

Abstract

A new notion of bent sequences was introduced in [1] as a solution in X, Y to the system

$$\frac{1}{\sqrt{n}} HX = Y,$$

where H is a real Hadamard matrix of order n and $X, Y \in \{\pm 1\}^n$. X is called *a bent sequence for H*. If H is the Sylvester Hadamard matrix then any bent Boolean function $f\colon \mathbb{Z}_2^n \to \mathbb{Z}_2$ determines a bent sequence for H by the rule $X = (-1)^f$ (and vice versa).

Clearly, the vector Y can also be shown to be a bent sequence attached to H^T, called the dual of X. When $X = Y$ the sequence X is said to be *self-dual.* In [2] this notion of self-dual bent sequence for a real Hadamard matrix was further generalized to a $n \times n$ Butson Hadamard matrix with entries in the set of complex 4-*th* roots of unity as a solution in X to the system

$$HX = \lambda X \tag{1}$$

where λ is an eigenvalue of H and $X \in \{\pm 1, \pm\sqrt{-1}\}^n$.

In this talk, we extend the definition of self-dual bent sequence X for H to any Butson Hadamard matrix (not only for the 4-*th* roots of unity) which is "complementary" to the definition given in [2]. That consists of considering, instead of (1), the system

$$\frac{1}{\sqrt{n}} HX = \overline{X} \quad \text{(or more generally, } HX = \lambda \overline{X}) \tag{2}$$

where the overline denotes complex conjugation, the entries of H and X belong to the set of complex k^{th} roots of unity. A solution X of the system (2) is what we understand in this work to be a self-dual bent sequence for a Butson Hadamard matrix H. Furthermore, when H and X take values in the set $\{\pm 1\}$, we recover the definition of [1]. Finally, it is easy to realize that if H is the complex conjugation of the m^{th} Kronecker power of the $q \times q$ Fourier matrix then any self-dual bent sequence for H determines a self-dual generalized bent function $f\colon \mathbb{Z}_q^m \to \mathbb{Z}_q$ by the rule $X = [\zeta_q^{f(\mathbf{a})}]^\top_{\mathbf{a} \in \mathbb{Z}_q^m}$ which we denote by $X = \zeta_q^f$ for convenience.

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A Framework for Classifying Cocyclic Hadamard Matrices of order $8p$

Santiago Barrera-Acevedo

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Abstract

We provide an update on ongoing work about the classification of cocyclic Hadamard matrices (CHMs) of order $8p$ with $p \geq 3$ prime. CHMs are Hadamard matrices whose entries are controlled by a 2-cocycle, and thus possess additional algebraic structure. It is conjectured that CHMs exist for all orders $4n$ where n is a positive integer. Ó Cathaín and Röder reported the classification of CHMs of orders less than 40. Barrera Acevedo et al., described an algorithm for classifying CHMs of orders $4p$ with $p > 3$ prime, up to equivalence, and classified such matrices for $p \leq 13$. In this talk, we discuss a framework for classifying CHMs of order $8p$ with $p \geq 3$ prime.

New constructions of higher dimensional Hadamard matrices and SBIBDs

Vedran Krčadinac

University of Zagreb

(Joint work with Mario Osvin Pavčević and Kristijan Tabak)

Abstract

Higher dimensional Hadamard matrices were introduced by Paul J. Shlichta [4, 5]. Warwick de Launey [1, 2] developed a framework for n-dimensional combinatorial designs of various kinds, including SBIBDs, Hadamard matrices, and their generalisations. We shall present several new constructions of such objects. A construction from [3] gives n-dimensional SBIBDs and proper regular Hadamard matrices with inequivalent slices. Another constructions gives three-dimensional Hadamard matrices of orders $v \equiv 2 \pmod 4$ which had previously been unknown.

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The Hadamard maximal determinant problem

PADRAIG Ó CATHÁIN

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(Joint work with Patrick Browne, Ronan Egan, Fintan Hegarty and Guillermo Nunez Ponasso)

Abstract

Hadamard's famous paper of 1893 discusses complex matrices which meet his bound with equality. Slightly less well known is the body of work on matrices with entries in -1,1 with maximal determinant. These are typically not Hadamard, since the Hadamard bound cannot be achieved when the dimension is larger than 2 and not a multiple of 4. I will survey the main techniques, bounds and constructions to be found in the literature, highlighting recent progress.

While the analogous complex Hadamard matrices (particularly with kth roots as entries) have been well studied, much less is known about complex maximal determinant matrices (over a fixed finite extension of the rationals) when the bound is not attained. I will present some open questions and directions for future research.

The chromatic number of some generalized Kneser graphs

Jozefien D'haeseleer

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(Joint work with Klaus Metsch and Daniel Werner)

Abstract

A set F of subspaces of $\mathrm{PG}(n, q)$ such that for all $\pi, \tau \in F$ we have $\pi \subseteq \tau$ or $\tau \subseteq \pi$ is called a *flag* and $\{\dim(\pi) : \pi \in F\}$ is called its *type*. Note that we work with projective dimensions. Furthermore, two flags F and F' are said to be in *general position*, if for all $\pi \in F$ and $\pi' \in F'$ we have $\pi \cap \pi' = \emptyset$ or $\langle \pi, \pi' \rangle = \mathrm{PG}(n, q)$.

For $\Omega \subseteq \{0, 1, \ldots, n-1\}$ we define the q-Kneser graph $\Gamma = qK_{n;\Omega}$ to be the graph whose vertices are all flags of type Ω of $\mathrm{PG}(n, q)$ with two vertices adjacent when the corresponding flags are in general position. Furthermore, a *co-clique* or *independent set* of Γ, that is, a set of vertices any two of which are non-adjacent, is called an *Erdős-Ko-Rado-set* (or, in short, *EKR-set*) in reference to the authors who first introduced this type of problem in set theory. Finally, a set of pairwise disjoint EKR-sets such that the union of these EKR-sets comprises all flags of Γ is called a *coloring* of Γ and the cardinality of a coloring of minimal size is called the *chromatic number* χ of Γ.

We are interested in the chromatic number of these Kneser graphs and hence in their independence number. In many cases these numbers are known, when $|\Omega| = 1$, $\Omega = \{k\}$ and $n \geq 2k + 2$. We investigated several Kneser graphs $qK_{n;\Omega}$ with $|\Omega| = 2$. Our main result gives the chromatic number of the q-Kneser graphs $qK_{4;\{1,3\}}$ and $qK_{2d;\{d-1,d\}}$, for $d = 2, 3$: $\chi(qK_{4;\{1,3\}}) = q^3+q^2+q+1$ for $q \geq 3$, and $\chi(qK_{4;\{1,2\}}) = q^3+q^2+1$, $\chi(qK_{6;\{2,3\}}) = q^4+q^3+q^2+1$, both for q large enough. We also give the structure of a minimum coloring of these graphs.

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Minimum weight of the code from intersecting lines in $\mathrm{PG}(3, q)$

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Polytechnic University of Catalonia - Department of Mathematics

(Joint work with Sam Adriaensen, Mrinmoy Datta and Leo Storme)

Abstract

Consider the linear code $\mathcal{C}$ defined as follows, where $q = p^h$, p prime. Let G be the matrix whose rows and columns are indexed by the lines of $\mathrm{PG}(3, q)$, with

$$(G)_{\ell_1 \ell_2} = \begin{cases} 0 & \text{if } \ell_1 \cap \ell_2 = \emptyset, \\ 1 & \text{if } \ell_1 \cap \ell_2 \neq \emptyset. \end{cases}$$

Then $\mathcal{C}$ is defined as the $\mathbb{F}_p$-vector space spanned by the rows of G.

The code $\mathcal{C}$ has length $(q^2 + 1)(q^2 + q + 1)$ (the number of lines in $\mathrm{PG}(3, q)$). In [2], it was shown that $\mathcal{C}$ has dimension $\frac{1}{6}p(2p^2 + 1)h + 1$. However, the minimum weight of $\mathcal{C}$ is still unknown. This open problem is a special case of [1, Open Problem 2.4].

In this talk, I will show how we can determine the minimum weight of $\mathcal{C}$.

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On a Problem Involving Strongly Orthogonal Roots

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Abstract

The notion of strongly orthogonal roots first appeared in the classical works of Harish-Chandra in the study of holomorphic discrete series representations and of Kostant in the study of conjugacy classes of real Cartan subalgebras. It has since appeared in many articles on different fields.

Fixing a natural number k and a root system R, we examine the maximal number of sets of k mutually strongly orthogonal roots so that any two such distinct sets have the property that the difference between the respective sums of all elements can itself be written as a sum of k roots that are mutually strongly orthogonal. The question that we address is derived from the open problem of (non-)existence of finite projective planes, which can be interpreted as belonging to the root system of type A. We formulate the general problem for all root systems and provide results in certain cases.

Some Results on Partial Difference Sets

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(Joint work with Stefaan De Winter, Ellen Kamischke and Eric Neubert)

Abstract

A few years ago we proved a theorem for strongly regular graphs that provides numerical restrictions on the number of fixed vertices and the number of vertices mapped to adjacent vertices under an automorphism. We then used this result to develop some new techniques to study regular partial difference sets in abelian groups. We have proved several non-existence results and classification results of partial difference sets in abelian groups. Also we completely answered the question "For which odd positive integer $v > 1$, can we find a Paley type partial difference set in an abelian group of order v?", a classical question from the 1990s.

In this talk I plan to give an overview of the main ideas used in our proofs and state our main results. I will conclude the talk with some ongoing research, and ideas for future research.

On the additivity of 2-(v, k, λ) designs

ANAMARI NAKIĆ

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(Joint work with Marco Buratti)

Abstract

A 2-(v, k, λ) design is *additive* under a commutative group G - or briefly G-additive - if its points can be injectively labeled with elements of G in such a way that every block has *weight* zero where the weight of a block is the sum of the labels of its points. In particular, it is *strongly* G-additive if its block set is precisely the set of zero-weight k-subsets of the point set. This interesting topic was introduced by Caggegi, Falcone and Pavone in [2].

One of the most challenging problems on additive designs is the construction of additive Steiner 2-designs whose block size k is neither a prime power greater than 2 nor a prime power plus one.

In the first part of my talk I will summarize how in [1] we have been able to solve this problem when k is neither singly even nor of the form $2^n 3$. Unfortunately our solution is not very satisfactory since the corresponding values of v are huge.

In the second part I will present some new results that we are obtaining in a long-term work still in progress:

(1) The design $\mathrm{PG}_d(n, q)$ of points and d-dimensional subspaces of $\mathrm{PG}(n, q)$ is additive under the elementary abelian group of order q^{n+1}.

(2) If p is a prime, the design $\mathrm{PG}_1(n, p)$ of points and lines of $\mathrm{PG}(n, p)$ is strongly $\mathbb{Z}_p^v$-additive where $v = \frac{p^{n+1}-1}{p-1}$ is the number of points of $\mathrm{PG}(n, p)$.

(3) Every cyclic 2-$(v, k, \lambda; n)$ symmetric design with $\gcd(n, v) = 1$ is $\mathbb{Z}_p^t$-additive for any pair (p, t) where p is a prime dividing n and t is its order in $U(\mathbb{Z}_v)$.

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Burning Steiner triple systems

Andrea Burgess

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(Joint work with Caleb Jones and David Pike)

Abstract

The concept of graph burning was introduced by Bonato, Janssen and Roshanbin as a model of information spread in a social network. At each time step, an arsonist sets fire to a vertex of a graph; simultaneously, existing fires spread to neighbouring vertices. A graph's burning number is the minimum number of vertices that the arsonist must set on fire so that all vertices burn. Graph burning is a dynamic area of study, and much work is ongoing towards proving the Burning Number Conjecture, which posits that the burning number of a graph of order n is at most $\lceil \sqrt{n} \rceil$.

In this talk, we extend the graph burning model to hypergraphs. We also introduce lazy burning, in which the arsonist's involvement is restricted to one turn where they can set fire to any number of vertices. After discussing a few general bounds on the hypergraph burning number, we focus on burning Steiner triple systems. Among other results, we show that for any integer $n \geq 3$, there is a Steiner triple system with lazy burning number n. Moreover, there is an STS(v) with lazy burning number 3 for all admissible orders v.

Extensions of Steiner Loops of Projective Type

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(Joint work with G. Falcone, A. Figula)

Abstract

This work sheds light on the connection between Steiner triple systems and commutative loops, and offers a classification approach using cohomology-inspired methods. Although it has been known since 50's of the last century ([6], [1]) that the operation $a \cdot b = c$ for any triple $\{a, b, c\}$ in a Steiner triple system $\mathcal{S}$ (together with $a \cdot a = \Omega \cdot \Omega = \Omega$ and $a \cdot \Omega = \Omega \cdot a = a$, for a further element Ω not in $\mathcal{S}$) gives in turn a commutative loop, an extension theory for Steiner triple systems has rarely been considered.

In this work, on the one hand we deal with non-central extensions of normal subloops using the so called *Steiner operator*.

On the other hand, in this framework of extension theory, we specifically focus on the case where the normal subloop is central, resulting in a *Shcreier extension.* This method provides a constructive approach to describing Steiner triple systems containing Veblen points.

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Harmonious coloring of the incidence graph of a design

Francesca Merola

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Abstract

A harmonious coloring of a graph is a proper vertex-coloring such that every pair of colors appears on at most one pair of adjacent vertices, and the harmonious chromatic number $\chi_H(G)$ of a graph G is then the minimum number of colors needed for a harmonious coloring of G.

It is easy to note that the harmonious chromatic number of the incidence graph of a 2-(v, k, λ)-design is bounded below by v, the number of points. I will present some examples and constructions of designs having harmonious chromatic number of the incidence graph exactly v, and discuss some connections between calculating this harmonious chromatic number and nesting of designs.

This talk is based on research still very much in progress started at a recent Banff workshop, and it is joint work with numerous authors.

Dual incidences arising from a subsets of spaces

Kristijan Tabak

Rochester Institute of Technology, Zagreb campus

Abstract

Let V be a n-dimensional vector space over $\mathbb{F}_q$ and $\mathcal{H}$ is any set of k-dimensional subspaces of V. We construct two incidence structures $\mathcal{D}_{max}(\mathcal{H})$ and $\mathcal{D}_{min}(\mathcal{H})$ using subspaces from $\mathcal{H}$. The points are subspaces from $\mathcal{H}$. The blocks of $\mathcal{D}_{max}(\mathcal{H})$ are indexed by all hyperplanes of V, while the blocks of $\mathcal{D}_{min}(\mathcal{H})$ are indexed by all subspaces of dimension 1. We show that $\mathcal{D}_{max}(\mathcal{H})$ and $\mathcal{D}_{min}(\mathcal{H})$ are dual in a sense that their incidence matrices are dependent, one can be calculated from the other. Additionally, if $\mathcal{H}$ is a $t-(n, k, \lambda)_q$-design we prove new matrix equations for incidence matrices of $\mathcal{D}_{max}(\mathcal{H})$ and $\mathcal{D}_{min}(\mathcal{H})$.

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On the diameter and zero forcing number of some graph classes in the Johnson, Grassmann and Hamming association scheme

Sjanne Zeijlemaker

Eindhoven University of Technology - Department of Mathematics and Computer Science

(Joint work with Aida Abiad, Robin Simoens)

Abstract

Graph classes in the Johnson, Grassmann and Hamming association scheme have received a considerable amount of attention over the last decades. Although several (NP-hard) graph parameters have been investigated for these families, many remain unknown. In this talk, we establish the diameter of generalized Grassmann graphs, extending previous results for generalized Johnson graphs. We also study the zero forcing number of generalized Johnson and Grassmann graphs, as well as Hamming graphs. As a corollary, we obtain the known results for Kneser graphs, Johnson graphs on 2-sets, lattice graphs and hypercubes. This is joint work with Aida Abiad and Robin Simoens.

Distance-regular graphs with classical parameters which support a uniform structure: case $q \leq 1$ (Part 1)

Roghayeh Maleki

University of Primorska, UP FAMNIT, Koper, Slovenia

(Joint work with Blas Fernández, Štefko Miklavič, and Giusy Monzillo)

Abstract

Let G be a connected bipartite graph. Then, its adjacency matrix A can be decomposed as $A = L + R$, where $L = L(x)$ and $R = R(x)$ are respectively the lowering and the raising matrices with respect to a certain vertex x. The graph G has a *uniform structure* with respect to x if the matrices RL^2, LRL, L^2R, and L satisfy a certain linear dependency.

Let $\Gamma = (X, E)$ be a connected non-bipartite graph. Fix a vertex $x \in X$ and let $\Gamma_f = (X, E_f)$ be the bipartite graph, where $E_f = E \setminus \{yz \mid \partial(x, y) = \partial(x, z)\}$ and ∂ is the distance function in Γ. The graph Γ is said to support a uniform structure whenever Γ_f has a uniform structure with respect to x. Assume that Γ is a non-bipartite distance-regular graph with classical parameters (D, q, α, β). It turns out that q is an integer different from 0 and -1.

In this talk, I will present the complete classification of non-bipartite distance-regular graphs with classical parameters (D, q, α, β) for $q \leq 1$ and $D \geq 4$, that support a uniform structure.

Distance-regular graphs with classical parameters that support a uniform structure: case $q \geq 2$ (Part 2)

Giusy Monzillo

University of Primorska - FAMNIT

(Joint work with B. Fernández, R. Maleki, and Š. Miklavič)

Abstract

With reference to the contribution of R. Maleki, this talk will illustrate whenever a 1-thin distance-regular graph Γ with classical parameters (D, q, α, β), $D \geq 4$ and $q \geq 2$, supports a uniform structure (w.r.t. a fixed vertex of Γ). By [2] and [3], in order that the latter property is satisfied, such a graph Γ must admit exactly two (thin) irreducible T-modules with endpoint 1 (one with diameter $D-2$ and the other with diameter $D-1$), up to isomorphism. The analysis which arises from this consideration shows that for $\alpha \neq 0$ there remain only two feasible infinite families, whose respective classical parameters are

$$\Big(D, q, q, \frac{q^2(q^D-1)}{q-1}\Big), \quad \text{with } D \text{ even}$$

$$\Big(D, q, q+1, \frac{q^{D+1}(q+1)-q^2-1}{q-1}\Big), \quad \text{with } D \text{ odd.}$$

Concerning the case $\alpha = 0$, examples are dual polar graphs, which are known to support a uniform structure [4, Proposition 26.4(i)]. Additionally assuming that Γ is a regular near polygon, it follows from [1, Theorem 9.4.4] that Γ is in fact a dual polar graph.

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On (non)symmetric association schemes and associated family of graphs

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Andrej Marušič Institute, University of Primorska

(Joint work with Giusy Monzillo)

Abstract

Let $\mathcal{M}$ denote the Bose-Mesner algebra of a commutative d-class association scheme $\mathfrak{X}$ that does not need to be symmetric. In this talk, we consider the following question: what combinatorial structure does (un)directed graph need to have so that its adjacency matrix will generate the Bose-Mesner algebra of a commutative d-class association scheme $\mathfrak{X}$?

Wednesday July 5

9:00-9:55 **Hadi Kharaghani: Balanced designs related to projective planes**

10:00-10:30 Coffee break

ROOM O-027

10:30-10:50 Ian Wanless: Relations on nets and MOLS

10:55-11:15 Raúl M. Falcón: The Hadamard quasigroup product of orthogonal Latin squares

11:20-11:40 Marco Buratti: Meet my favorite net

11:45-12:05 Charlene Weiß: Existence of small ordered orthogonal arrays

ROOM O-S31

10:30-10:50 Alena Ernst: Erdős-Ko-Rado theorems for finite general linear groups

10:55-11:15 Sarobidy Razafimahatratra: An Erdős-Ko-Rado theorem for transitive groups of degree a product of two odd primes

11:20-11:40 Yulia Kempner: Violator Spaces and Greedoids

11:45-12:05 Habibul Islam: Construction of Galois LCD MDS Codes

Balanced designs related to projective planes

Hadi Kharaghani
University of Lethbridge
(Joint work with Sho Suda)

Abstract

A link between projective planes and balancedly splittable partial Hadamard matrices will be introduced. The applications include the existence of splittable balanced incomplete designs, maximal equiangular lines and Hadamard matrices with large or maximal excess.

Relations on nets and MOLS

Ian Wanless
Monash University - School of Mathematics
(Joint work with Michael Gill)

Abstract

A k-net is a geometry equivalent to $(k-2)$ Mutually Orthogonal Latin Squares (MOLS). A *relation* is a linear dependence in the point-line incidence matrix of the net. In 2014 Dukes and Howard showed that any 6-net of order 10 satisfies at least two non-trivial relations. This opens up a possibile avenue towards showing the non-existence of 4 MOLS of order 10. We generated all 4-nets of order 10 that satisfy a non-trivial relation and also ruled out one type of relation on 5-nets. I will discuss these computations, as well as some of the theory of relations on nets more generally.

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The Hadamard quasigroup product of orthogonal Latin squares

Raúl M. Falcón

Universidad de Sevilla - Department of Applied Mathematics I

(Joint work with V. Álvarez, J.A. Armario, M.D. Frau, F. Gudiel, M.B. Güemes and L. Mella.)

Abstract

Let $\mathcal{A}(n)$ and $\mathcal{L}(n)$ denote, respectively, the set of $n \times n$ arrays, and the set of Latin squares of order n, all of them with entries in the set $[n] := \{1, \ldots, n\}$. Let $A, B \in \mathcal{A}(n)$ and $L \in \mathcal{L}(n)$. As a natural generalization of the classical Hadamard product, the *Hadamard quasigroup product* $A \odot_L B \in \mathcal{A}(n)$ has recently been introduced [1] so that

$$(A \odot_L B)\,[i,j] := L\,[A[i,j], B[i,j]]\,, \text{ for all } i,j \in [n]. \tag{3}$$

Let $\mathcal{OL}(n)$ denote the set of pairs of orthogonal Latin squares in $\mathcal{L}(n)$. In this talk, we are interested in studying under which conditions $L_1 \odot_{L_3} L_2 \in \mathcal{L}(n)$, for $(L_1, L_2) \in \mathcal{OL}(n)$ and $L_3 \in \mathcal{L}(n)$. It requires the existence of an involution

$$\varphi : \mathcal{OL}(n) \to \mathcal{OL}(n)$$

$$(L_1, L_2) \to (\varphi^{\rho}_{L_2}(L_1), \varphi^{\ell}_{L_1}(L_2))$$

where $\begin{cases} \varphi^{\rho}_{L_2}(L_1)\,[L_1[i,j],\, L_2[i,j]] := i, \\ \varphi^{\ell}_{L_1}(L_2)\,[L_1[i,j],\, L_2[i,j]] := j. \end{cases}$, for all $i, j \in [n]$.

Theorem 1. *The following statements hold.*

a) $L_1 \odot_{L_3} L_2 \in \mathcal{L}(n)$ *if and only if* $\varphi^{\rho}_{L_2}(L_1)$, $\varphi^{\ell}_{L_1}(L_2)$ *and* L_3 *are MOLS.*

b) L_1, L_2 *and* L_3 *are MOLS if and only if* $\varphi^{\rho}_{L_2}(L_1)$, $\varphi^{\ell}_{L_1}(L_2)$ *and* $\varphi^{\rho}_{L_2}(L_1) \odot_{L_3} \varphi^{\ell}_{L_1}(L_2)$ *are MOLS.*

Based on this theorem, we describe illustrative examples showing how the involution φ establishes a new way to connect distinct species of sets of three MOLS.

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Meet my favorite net

Marco Buratti

Sapienza University of Rome

(Joint work with Anita Pasotti)

Abstract

A (r, n)-*net* is a partial linear space with n^2 points and rn lines, each of size n, arranged into r *parallel classes*. Speaking of a *Heffter* (r, n)-*net* over a group G, we mean a (r, n)-net whose points form a *half-set* of G and whose lines are all zero-sum in G. This terminology is justified by the fact that a Heffter $(2, n)$-net over $\mathbb{Z}_{2n^2+1}$ is essentially the same as what is called a *Heffter array* $\mathrm{H}(n, n)$ in the literature.

Constructing Heffter (r, n)-nets with $r > 2$ seems to be difficult and even a great challenge for $r > 3$. After much effort we have been able to get Heffter $(3, n)$-nets for infinitely many values of n by means of recursive constructions and some "magic" tools. On the other hand, until recently, we did not even have an example of a Heffter net with 4 parallel classes. So it was a plot twist when we came across one with 9 parallel classes. It is a Heffter $(9, 11)$-net over $\mathbb{Z}_3^5$. In this talk I will retrace the path that led us to this amazing net.

Existence of small ordered orthogonal arrays

Charlene Weiß

Paderborn University - Department of Mathematics

(Joint work with Kai-Uwe Schmidt)

Abstract

Ordered orthogonal arrays generalize orthogonal arrays and have numerous applications, in particular in coding theory, cryptography, and numerical integration via their connection to (t, m, s)-nets. The main question is the existence of ordered orthogonal arrays having as few rows as possible. A lower bound on the minimum number of rows in an ordered orthogonal array is given by the famous Rao bound for orthogonal arrays. We show that there exist ordered orthogonal arrays, whose sizes deviate from this Rao bound by a factor that is polynomial in the parameters of the ordered orthogonal array. The proof is nonconstructive and based on a probabilistic method due to Kuperberg, Lovett, and Peled.

Erdős-Ko-Rado theorems for finite general linear groups

Alena Ernst

Paderborn University - Department of Mathematics

(Joint work with Kai-Uwe Schmidt)

Abstract

We call a subset Y of the finite general linear group $\mathrm{GL}(n, q)$ *t-intersecting* if $\mathrm{rk}(x-y) \leq n-t$ for all $x, y \in Y$. In this talk we give upper bounds on the size of t-intersecting sets and characterise the extremal cases that attain the bound. This is a q-analog of the corresponding result for the symmetric group, which was conjectured by Deza and Frankl in 1977 and proved by Ellis, Friedgut, and Pilpel in 2011. The results are obtained by using eigenvalue techniques and the theory of association schemes plays a crucial role.

An Erdős-Ko-Rado theorem for transitive groups of degree a product of two odd primes

Sarobidy Razafimahatratra

University of Primorska - FAMNIT

(Joint work with Angelot Behajaina, Roghayeh Maleki and Karen Meagher)

Abstract

Given a finite transitive group $G \leq \mathrm{Sym}(\Omega)$, a set $\mathcal{F} \subset G$ is *intersecting* if for any $g, h \in G$, there exists $\omega \in \Omega$ such that $\omega^g = \omega^h$. The *intersection density* $\rho(G)$ is the maximum ratio of $\frac{|\mathcal{F}|}{|G_\omega|}$, where $\mathcal{F}$ runs through all intersecting sets of G and G_ω is the stabilizer of $\omega \in \Omega$ in G.

In [1], it was conjectured that any transitive group of degree a product of two distinct odd primes $p > q$ has intersection density equal to 1. This was disproved by Marušič et al. in [2] by constructing an imprimitive group of degree pq with a block system consisting of blocks of size q, and whose intersection density is q.

In this talk, I will present some recent results ([3, 4]) on the intersection density of transitive groups of degree a product of two distinct odd primes $p > q$. In particular, I will focus on primitive groups and imprimitive groups with a unique block system, whose blocks have size q.

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Violator Spaces and Greedoids

Yulia Kempner
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(Joint work with Vadim E. Levit)

Abstract

The primary objective of this presentation is to establish connections between two prominent, yet earlier independently developed theories: the theory of violator spaces and the theory of greedoids.

Violator spaces, initially proposed by Matoušek et al. [1] in 2008, serve as a generalization of linear programming problems. Originally, violator spaces were defined for the set of constraints denoted as H, where each subset $A \subseteq H$ corresponds to a set $V(A)$ consisting of all constraints violating by A.

For example, a violator space naturally emerges when determining the smallest enclosing ball of a finite set of points in R^d. In this context, the set H represents a collection of points in R^d, and the violated constraints of a given subset of points A are precisely the points lying outside the smallest enclosing ball of A. If we consider the points situated within the enclosing ball, we obtain a weakened version of a closure operator.

In our investigation, we explore the interrelations between violator spaces and closure spaces and demonstrate that the family of closure spaces can be regarded as a sub-family of violator spaces [2].

Greedoids, introduced by Korte and Lovász [3] in 1981, aim to characterize combinatorial structures wherein greedy algorithms yield optimal solutions. In our study, we establish that the family of greedoids also falls within the realm of violator spaces.

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Construction of Galois LCD MDS Codes

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(Joint work with Anna-Lena Horlemann)

Abstract

For a prime q, let $\mathbb{F}_{q^m}$ be the field of q^m elements. For $0 \leq e \leq m-1$, the e-Galois inner product (which generalizes both Euclidean and the Hermitian product) was introduced in 2017 as

$$\langle \boldsymbol{u}, \boldsymbol{v} \rangle_e := \sum_{i=1}^{n} u_i v_i^{q^e}$$

for $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{F}_{q^m}^n$. Accordingly, the e-Galois dual of $C \subseteq \mathbb{F}_{q^m}^n$ is defined as

$$C^{\perp_e} := \{\boldsymbol{v} \in \mathbb{F}_{q^m}^n \mid \langle \boldsymbol{u}, \boldsymbol{v} \rangle_e = 0 \text{ for all } \boldsymbol{u} \in C\}.$$

A linear code C of length n is said to be linear complementary dual (shortly, LCD) if $C \cap C^{\perp_e} = \{0\}$. Exploring new LCD MDS (maximum distance separable) codes has been an important task due to their efficient applications in cryptography [1] and quantum codes [5]. The constructions of Euclidean and Hermitian LCD codes are explicitly known by [2]. In this article, we explicitly construct three new classes of e-Galois LCD MDS codes which have not been discovered so far. We consider Generalized Reed-Solomon (GRS) codes and obtain two classes of e-Galois LCD codes, first of length $n = q^s$ where $1 \leq s \leq m$, and second of length $n \leq q^m$ with dimension $k = q^m - n$, respectively. Further, by using Extended GRS codes, we also explore e-Galois LCD codes of length $q^m + 1$.

We remark that the authors of [3] investigated σ-LCD codes where the map σ defines a more general product called σ-inner product. In particular, they proved that a linear code $[n, k]$ is equivalent to a σ-LCD code. However, σ can never be the e-Galois product, except in the trivial Euclidean case. Therefore, our obtained codes are not covered by [3]. Furthermore, our constructions are comparatively explicit and compact.

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Thursday July 6

9:00-9:55 **Leo Storme: Applications of finite geometries**

10:00-10:30 Coffee break

ROOM O-027

10:30-10:50 Stefano Lia: A geometrical picture: semifields and non-singular sublines

10:55-11:15 Sam Adriaensen: On the minimum size of linear sets

11:20-11:40 Stefaan De Winter: Projective Two-Weight Sets

ROOM O-S31

10:30-10:50 Daniel Panario: Trade-Based LDPC Codes

10:55-11:15 Clementa Alonso-González: Motzkin Numbers and Flag Codes

11:20-11:40 Ivica Martinjak: Refined Enumeration of the Catalan Family of Alternating Sign Matrices

11:45-12:05 Dominik Beck: Fourth Moment of Random Determinant

14:30-15:25 **Aida Abiad: On eigenvalue bounds for the independence and chromatic number of graph powers and its applications**

15:30-16:00 Coffee break

ROOM O-027

16:00-16:20 Antonina P. Khramova: Sum-rank-metric graphs and eigenvalue bounds for network coding

16:25-16:45 Jesse Lansdown: Rank 3 graphs and the Delsarte and Hoffman bounds

16:50-17:10 Alexander Gavrilyuk: On strongly regular graphs decomposable into a divisible design graph and a coclique

17:15-17:35 Robert Bailey: Block-colourings of star systems

ROOM O-S31

16:00-16:20 Lucia Moura: Cover-free families on hypergraphs

16:25-16:45 Amruta Shinde: Another generalization for measure of fault tolerance in hypercubes

16:50-17:10 Nancy E. Clarke: Reconfiguration for Dominating Sets

17:15-17:35 Nino Basic: A relation between vertex and edge orbits in nut graphs

Applications of finite geometries

Leo Storme

Ghent University

Abstract

Within finite geometries, a great variety of substructures are investigated. A lot of these substructures are investigated because of their geometrical importance, but, often, these substructures are of great relevance for problems in other research domains.

Classical examples include many links with coding theory: linear MDS codes and arcs, linear codes meeting the Griesmer bound and minihypers, ..., [1].

But there are also links with graph theory, algebraic combinatorics, cryptography, Latin squares and Sudoku's, and even logic [2].

The nice fact is that the number of links with other research domains keeps increasing. A recent example are the links of finite geometries with subspace codes and random network coding [1].

This talk will present a number of links of substructures of finite geometries to other research domains.

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A geometrical picture: semifields and non-singular sublines.

Stefano Lia

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(Joint work with John Sheekey)

Abstract

Building on the representation of three-fold tensors as points of $\mathrm{PG}(3, q^2)$, we exploit a geometrical framework allowing us to provide an interesting geometrical interpretation of the non-singularity of tensors. As a consequence, constructions of new quasi-hermitian surfaces, classifications of non-singular four-fold tensors, and new results on semifields (incorporating a new geometrical proof of a classical result) are obtained.

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On the minimum size of linear sets

Sam Adriaensen
Vrije Universiteit Brussel - Department of Mathematics and Data Science
(Joint work with Paolo Santonastaso)

Abstract

A *linear set* in the projective space $\mathrm{PG}(d, q)$ is a set of points which is not (necessarily) a subspace, but does arise from a subspace of $\mathbb{F}_q^{d+1}$ over a subfield of $\mathbb{F}_q$. Linear sets have gained interest in recent years due to their connection with many combinatorial objects such as blocking sets, KM-arcs, and rank-metric codes. De Beule and Van de Voorde [2] gave a tight lower bound on the size of a linear set in $\mathrm{PG}(1, q)$, and generalised this to a bound on the size of a linear set in higher dimensions, intersecting some hyperplane in a subgeometry. We generalise this to a lower bound on the size of a linear set meeting some subspace (not necessarily a hyperplane) in a subgeometry. We also discuss some constructions of linear sets attaining equality in this bound. This talk is based on [1].

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Projective Two-Weight Sets

Stefaan De Winter
The National Science Foundation

Abstract

In this talk we will describe the construction of a class of projective two-weight sets in $\mathrm{PG}(3n-1, q)$, $n > 1$, that yield strongly regular graphs with the same parameters as those that arise from maximal arcs in $\mathrm{PG}(2, q^n)$. Our construction works for both even and odd q and is of particular interest when q is odd as it is known that in that case no maximal arcs exist in $\mathrm{PG}(2, q^n)$.

Trade-Based LDPC Codes

Daniel Panario

Carleton University - School of Mathematics and Statistics

(Joint work with F. Amirzade and M.-R. Sadeghi)

Abstract

We provide [1] a novel approach to construct the parity-check matrix of an LDPC (Low-Density Parity-Check) code based on trades obtained from block designs. We call these codes *trade-based LDPC codes*. Using properties of cyclical trades, we consider the graphical structures of the Tanner graph of these codes such as short cycles, girth, as well as trapping sets. These are key factors when determining the minimum distance of these codes.

It is known [2] that the minimum distance of single-edge protograph-based LDPC codes, such as single-edge Quasi-cyclic LDPC (QC-LDPC) codes whose parity-check matrices have column weight m, is upper bounded by $(m+1)!$. One of the merits of trade-based LDPC codes over some different types of LDPC codes is that the minimum distances of the trade-based LDPC codes exceed the mentioned upper bound limit for single-edge LDPC codes.

Multi-edge QC-LDPC codes have potentially larger minimum distances than single-edge QC-LDPC codes [3]. However, studying features of their Tanner graph is harder than for single-edge QC-LDPC codes. Using the parity-check matrix of trade-based LDPC codes, we present a method to define base matrices of multi-edge QC-LDPC codes. The construction of exponent matrices corresponding to these base matrices has less complexity than the proposed in the literature. We prove that these base matrices result in QC-LDPC codes with smaller lower bounds on the lifting degree than existing ones.

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Motzkin Numbers and Flag Codes

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(Joint work with Miguel Ángel Navarro-Pérez)

Abstract

Motzkin numbers were introduced by T. Motzkin in [4] to count the number of ways of connecting n points on a circle by non-intersecting chords. Since then, the integer sequence of Motzkin numbers and other related sequences, have been widely studied since they count many different combinatorial objects (see [3]). In this talk we present a new appearance of this sequence in the coding theory context, more precisely, in the study of flag codes. A *full flag* is a sequence of nested $\mathbb{F}_q$-subspaces of dimensions $(1, \dots, n-1)$ of a vector space $\mathbb{F}_q^n$ and a *full flag code* is a nonempty subset of full flags. The *flag distance* is defined as the sum of the respective subspace distances and it can be described by *distance vectors* (see [2]). Each distance vector represents a different way of attaining a possible value of the flag distance. In [1], we show that the n-th Motzkin number counts the number of different distance vectors corresponding to full flags on $\mathbb{F}_q^n$. We also identify the integer sequence giving the number of possible distance vectors associated with a specific value of the flag distance and other related sequences that can be read in the context of full flag codes.

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Refined Enumeration of the Catalan Family of Alternating Sign Matrices

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(Joint work with Ana Mimica)

Abstract

Alternating sign matrices are matrices whose non-zero elements alternate in sign, and that sum to 1 per each row and column. In this paper, we extend the notion of permutation pattern to these matrices. We study a family of alternating sign matrices with permutation pattern avoidance and a constraint on relative positions of 1s among neighboring rows. Refined enumerations of these matrices with respect to the special element and with respect to position of $1s$ in the first row are provided. We also introduce further families of these matrices, that generalize permutation matrices.

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Fourth Moment of Random Determinant

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Abstract

Let X_{ij} be independent and identically distributed random variables, from which we construct matrices $A = (X_{ij})_{n\times n}$ and $U = (X_{ij})_{n\times p}$. We denote moments of their entries X_{ij} as $m_r = \mathbb{E}X_{ij}^r$ and their central moments as $\mu_r = \mathbb{E}\,(X_{ij} - m_1)^r$. Is there a way how we can express the even moments of determinants $\det A$ and $(\det U^\top U)^{1/2}$ in an exact form? That is, the objective is to find $f_k(n) = \mathbb{E}\,(\det A)^k$ and $f_k(n,p) = \mathbb{E}\,(\det U^\top U)^{k/2}$ as a function of m_r (or μ_r). Equivalently, one could first try to find the generating functions $F_k(t) = \sum_{n=0}^{\infty} \frac{t^n}{(n!)^2} f_k(n)$ and $F_k(t,\omega) = \sum_{n=0}^{\infty}\sum_{p=0}^{n} \frac{(n-p)!}{n!p!} t^p \omega^{n-p} f_k(n,p)$.

The exact expression for $F_2(t)$ can be easily derived using recurrences for any distribution of X_{ij}. For higher moments, it is not that simple. In the case of fourth moment, Nyquist, Rice and Riordan found the expression for $F_4(t)$ when $m_1 = 0$. Later, Dembo [2] derived $F_4(t,\omega)$ when $m_1 = 0$. The general case for both $F_4(t)$ and $F_4(t,\omega)$ when $m_1 \neq 0$ remained unsolved. However, as shown in recent arXiv preprint [1], we obtained

$$F_4(t) = \frac{e^{t(\mu_4 - 3\mu_2^2)}}{(1-\mu_2^2 t)^5}\left(1 + \sum_{k=1}^{6} p_k t^k\right) \quad \text{and}$$

$$F_4(t,\omega) = \frac{e^{t(\mu_4 - 3\mu_2^2)}}{(1-\mu_2^2 t)^4 (1-\omega-\mu_2^2 t)}\left(1 + \sum_{k=1}^{6} p_k t^k + \frac{\omega m_1^2}{1-\omega-\mu_2^2 t}\sum_{k=1}^{4} \tilde{p}_k t^k + \frac{2\omega^2 m_1^4 \mu_2^2 t^2}{(1-\omega-\mu_2^2 t)^2}\right),$$

where p_k and $\tilde{p}_k$ are constants depending on m_1 and μ_r as polynomials of low order (see [1] for their exact definition). From those generating functions, one can easily deduce the moments $f_4(n)$ and $f_4(n,p)$ via Taylor expansion.

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On eigenvalue bounds for the independence and chromatic number of graph powers and its applications

Aida Abiad

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Abstract

In this talk I will present several eigenvalue bounds on the independence number and the distance chromatic number of graph powers. We will see how to use polynomials and mixed-integer linear programming in order to optimize such bounds. Infinite families of graphs for which the new bounds are tight will be shown, and also some applications to quantum information theory and coding theory will be discussed.

Sum-rank-metric graphs and eigenvalue bounds for network coding

Antonina P. Khramova

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(Joint work with Aida Abiad and Alberto Ravagnani)

Abstract

This talk focuses on the properties of sum-rank-metric graphs, which we introduce and study in connection with a problem in coding theory.

The vertices of a sum-rank-metric graph are t-tuples of matrices $X = (X_1, \dots, X_t)$ of possibly different sizes with entries from a finite field. Matrix tuples X and Y are adjacent if their sum-rank distance, defined as the number $\sum_{i=1}^{t} \operatorname{rank}(X_i - Y_i)$, is equal to 1. It is known that the geodesic distance of this graph coincides with the sum-rank distance of matrix tuples. The latter plays a crucial role in measuring the correction capability of codes in the context of multi-shot network coding.

We establish combinatorial properties of sum-rank-metric graphs. We also show several new eigenvalue bounds on the cardinality of a sum-rank-metric code and we illustrate, both with sporadic examples and infinite families, that our eigenvalue bounds are sharp in some instances and outperform the best bounds currently available.

Rank 3 graphs and the Delsarte and Hoffman bounds

Jesse Lansdown

University of Canterbury - School of Mathematics and Statistics

(Joint work with John Bamberg, Michael Giudici and Gordon F. Royle)

Abstract

A graph is a *rank* 3 *graph* if its automorphism group has precisely one orbit on edges and precisely one orbit on non-edges. A classification of the rank 3 graphs follows from the classification of the *rank* 3 *groups* (groups with precisely 3 orbits on ordered pairs).

Rank 3 graphs are strongly regular and so the size of cliques and cocliques are bounded in terms of the graph's eigenvalues via the *Delsarte* and *Hoffman* bounds. Motivated by problems in the synchronisation hierarchy of permutation groups [1] we classify the rank 3 graphs which fail to meet at least one of the Delsarte or Hoffman bounds [2], up to some notoriously difficult cases.

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On strongly regular graphs decomposable into a divisible design graph and a coclique

Alexander Gavrilyuk

Shimane University

(Joint work with Vladislav Kabanov)

Abstract

We will discuss a generalization of the construction of strongly regular graphs, presented in [1]. It starts with a divisible design graph (which can be obtained from a variation of the Wallis – Fon-Der-Flaass prolific construction [2]) and extends it to a strongly regular graph by adding a coclique whose size is to satisfy the Hoffman-Delsarte bound.

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Block-colourings of star systems

Robert Bailey

Memorial University – Grenfell Campus

(Joint work with Iren Darijani)

Abstract

An *e-star system* of order n is a decomposition of a complete graph K_n into copies of a complete bipartite graph $K_{1,e}$. Necessary and sufficient conditions for the existence of e-star systems were obtained in the 1970s by Yamamoto et al., and for resolvable e-star systems in the 1990s by Yu. In the case where an e-star system exists but a resolvable system does not, it is natural to ask for the *chromatic index*, i.e. the minimum number of colours required to colour the blocks, of such a design. In this talk, we will give constructive upper bounds on the chromatic index of e-star systems when $n \equiv 0, 1 \bmod 2e$, for arbitrary $e \geq 3$, and in all cases for $e = 3$.

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Cover-free families on hypergraphs

Lucia Moura

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(Joint work with Thais Bardini Idalino)

Abstract

Cover-free families are widely studied combinatorial objects used in combinatorial group testing and in applications in cryptography and communications [3]. A d-CFF(t, n) is a $t \times n$ incidence matrix of a set system where no set is contained in the union of up to d other sets. Cover-free families are used for solving the non-adaptive group testing problem: find a set of up to d defective items among n items by testing them in pre-specified groups. A negative test indicates that all items in the group are non-defective, while a positive test shows that the group contains some defective items; the objective is to minimize the number t of tests while identifying all defective items.

In this talk, we consider cover-free families on hypergraphs, which are generalizations of cover-free families where possible sets of defective items are specified by the edges of a hypergraph. Various authors have recently considered this type of generalization in group testing [1, 2, 4]. We focus on constructions of cover-free families on hypergraphs and ongoing research initiated in [2].

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Another generalization for measure of fault tolerance in hypercubes

Amruta Shinde

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(Joint work with Uday Jagadale)

Abstract

Interconnection networks can be modeled by a connected graph with each processor in the network represented by a vertex, while the communication link between any two processors is represented by an edge. Hypercube is one of the most popular interconnection networks for parallel and distributed computing systems. We introduce a new measure for fault tolerance of graphs which combines conditional [2], component [1, 4] and structure [3] connectivities. Let G be a connected graph and $r \geq 2$, $h \geq 0$ be integers. Let S be a set of connected subgraphs of G such that every member of S is isomorphic to a connected subgraph H of G. Then S is called an h-conditional r-component H-structure cut of G, if there are at least r connected components in $G - V(S)$ and each component has minimum degree at least h. The h-conditional r-component H-structure connectivity of G is the minimum $|S|$ overall h-conditional r-component H-structure cut of G. In this paper, we investigate the h-conditional r-component H-structure connectivity of hypercube Q_n for $H \in \{Q_m : \ m < n\}$.

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Reconfiguration for Dominating Sets

Nancy E. Clarke

Acadia University

(Joint work with Kira Adaricheva, Heather Smith Blake, Chassidy Bozeman, Ruth Haas, Margaret-Ellen Messinger, and Karen Seyffarth)

Abstract

The dominating graph of a graph G has as its vertices all dominating sets of G, with two vertices adjacent if the corresponding dominating sets differ by the addition or deletion of a single vertex of G. We are interested in the properties of such graphs. In particular, we show that the dominating graph of any tree has a Hamilton path and that the dominating graph of a cycle on n vertices has a Hamilton path if and only if n is not a multiple of 4.

A relation between vertex and edge orbits in nut graphs

Nino Bašić

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(Joint work with Patrick W. Fowler and Tomaž Pisanski)

Abstract

A nut graph has a single non-trivial kernel eigenvector and that vector contains no zero entries. If the isolated vertex is excluded as trivial, nut graphs have seven or more vertices; they are all connected, non-bipartite, and have no leaves. A nut graph may be vertex transitive; there are known examples of circulant nut graphs, Cayley nut graphs, and also non-Cayley vertex-transitive nut graphs. We will show that no nut graph can be edge transitive. Furthermore, a nut graph always has strictly more edge orbits than vertex orbits. We also construct several families of nut graphs with a low number of vertex orbits and edge orbits as regular coverings over certain voltage graphs (using non-cyclic groups).

Friday July 7

9:00-9:55 **Ronan Egan: A survey of complex generalized weighing matrices, and a construction of quantum error-correcting codes**

10:00-10:30 Coffee break

10:30-10:50 Adrián Torres-Martín: Partial permutation decoding for $\mathbb{Z}_{p^s}$-linear generalized Hadamard codes

10:55-11:15 Nina Mostarac: s-PD-sets for codes from projective planes $\mathrm{PG}(2, 2^h)$, $5 \leq h \leq 9$

11:20-11:40 Sara Ban: Cyclic self-orthogonal $\mathbb{Z}_{2^k}$-codes constructed from generalized Boolean functions

11:45-12:05 Matteo Mravić: Some new extremal $\mathbb{Z}_4$-codes of lengths 32 and 40

14:30-15:25 **Vladimir D. Tonchev: Ternary self-dual codes, Hadamard matrices and related designs**

15:30-16:00 Coffee break

16:00-16:20 Bernardo Rodrigues: On two-weight codes invariant under the 3-fold covers of the Mathieu groups M_{22} and $\mathrm{Aut}(\mathrm{M}_{22})$

16:25-16:45 Tin Zrinski: Genetic algorithms in constructions of block designs and SRGs

16:50-17:10 Ana Šumberac: Codes from quasi-symmetric designs

17:15-17:35 Ana Grbac: Block designs from self-dual codes obtained from Paley designs and Paley graphs

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17:40-17:45 Closing

A survey of complex generalized weighing matrices, and a construction of quantum error-correcting codes

Ronan Egan

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Abstract

Let $\mathcal{U}_k$ denote the set of all complex k^{th} roots of unity. An $n \times n$ matrix W with entries in $\mathcal{U}_k \cup \{0\}$ is a *complex generalized weighing matrix* of *weight* w if $WW^* = wI_n$, where W^* denotes the complex conjugate transpose of W, and I_n denotes the $n \times n$ identity matrix. The set of all such matrices is denoted by $\mathrm{CGW}(n, w; k)$. Subsets of $\mathrm{CGW}(n, w; k)$ include weighing matrices ($k = 2$), Butson Hadamard matrices ($w = n$), and real Hadamard matrices (both $k = 2$ and $w = n$). These subsets have each received a large amount of attention due to their many applications and the interesting problems they present, but complex generalized weighing matrices in full generality have attracted comparatively little scrutiny.

This talk will be in two parts. First we will summarize a survey of the topic of complex generalized weighing matrices. We will discuss some of the known existence conditions, due mostly to de Launey [2], and constructions due mostly to Berman [1] and Seberry and Whiteman [3], that do not apply to any of these special subsets. Then to motivate this, we will demonstrate how complex generalized weighing matrices with appropriate parameters can be used to build Hermitian self-orthogonal codes over finite fields of square order, which can in turn be used to construct quantum error-correcting codes. Some early results of this type will be presented.

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Partial permutation decoding for $\mathbb{Z}_{p^s}$-linear generalized Hadamard codes [1]

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(Joint work with Josep Rifà and Mercè Villanueva)

Abstract

A code C over $\mathbb{Z}_p$ of length n is a nonempty subset of $\mathbb{Z}_p^n$, with p prime. A nonempty subset $\mathcal{C} \subseteq \mathbb{Z}_{p^s}^n$ is a $\mathbb{Z}_{p^s}$-additive code of length n if it is a subgroup of $\mathbb{Z}_{p^s}^n$. Let $\phi : \mathbb{Z}_{p^s} \longrightarrow \mathbb{Z}_p^{p^{s-1}}$ be the generalization of the usual Gray map, given in [2], and Φ its component-wise extension. We say that $C = \Phi(\mathcal{C})$ is a $\mathbb{Z}_{p^s}$-linear code of length np^{s-1}. A $\mathbb{Z}_{p^s}$-additive code $\mathcal{C}$ such that $C = \Phi(\mathcal{C})$ is a generalized Hadamard (GH) code is called a $\mathbb{Z}_{p^s}$-additive GH code and $C = \Phi(\mathcal{C})$ is called a $\mathbb{Z}_{p^s}$-linear GH code. Recall that a GH code over $\mathbb{Z}_p$ of length N has pN codewords and minimum distance $(p-1)N/p$.

In [3], it was shown that $\mathbb{Z}_{p^s}$-linear codes are systematic, by giving a systematic encoding. This makes $\mathbb{Z}_{p^s}$-linear codes suitable to apply the permutation decoding method. A partial permutation decoding can be executed if r-PD-sets, which are subsets of the permutation automorphism group of the code, are provided. We describe the permutation automorphism group of $\mathbb{Z}_{p^s}$-linear GH codes and show how to construct r-PD-sets of minimum size $r+1$, for all r up to an upper bound. A generalization of previous results [1] suitable for some families of $\mathbb{Z}_4$-linear Hadamard codes is provided, as well as new constructions that increase the value of r for a general class of $\mathbb{Z}_8$-linear GH codes.

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s-PD-sets for codes from projective planes $\mathrm{PG}(2, 2^h)$, $5 \leq h \leq 9$

Nina Mostarac

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(Joint work with Dean Crnković, Bernardo G. Rodrigues and Leo Storme)

Abstract

In this talk we will describe constructions of s-PD-sets for codes from certain projective planes, for $s = 2$ and $s = 3$. A construction of 2-PD-sets of 16 elements for codes from the Desarguesian projective planes $\mathrm{PG}(2, q)$, where $q = 2^h$ and $5 \leq h \leq 9$, will be given. We will also describe a construction of 3-PD-sets of 75 elements for the code from the Desarguesian projective plane $\mathrm{PG}(2, q)$, where $q = 2^9$. These 2-PD-sets and 3-PD-sets can be used for partial permutation decoding of codes obtained from the Desarguesian projective planes.

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Cyclic self-orthogonal $\mathbb{Z}_{2^k}$-codes constructed from generalized Boolean functions

Sara Ban

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(Joint work with Sanja Rukavina)

Abstract

A Boolean function on n variables is a mapping $f : \mathbb{F}_2^n \to \mathbb{F}_2$. A bent function is a Boolean function f such that $W_f(v) = \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x)+\langle v,x \rangle} = \pm 2^{\frac{n}{2}}$, for every $v \in \mathbb{F}_2^n$. A generalized Boolean function on n variables is a mapping $f : \mathbb{F}_2^n \to \mathbb{Z}_{2^h}$.

The subject of this talk is a construction of cyclic self-orthogonal codes over $\mathbb{Z}_{2^k}$ from generalized Boolean functions.

We give three constructions of cyclic self-orthogonal codes over $\mathbb{Z}_{2^k}$, for $k \geq 3$. In the first construction we start from a generalized Boolean function determined by three Boolean functions on n variables to obtain a self-orthogonal $\mathbb{Z}_{2^k}$-code of length 2^{n+2} with all Euclidean weights divisible by 2^{k+1}, for $3 \leq k \leq n$. In the second and the third construction, for every $k \geq 3$, we generate a self-orthogonal $\mathbb{Z}_{2^k}$-code of length 2^{n+1} with all Euclidean weights divisible by 2^{2k-1}, starting from a pair of bent functions on n variables.

Some new extremal $\mathbb{Z}_4$-codes of lengths 32 and 40

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(Joint work with Sanja Rukavina)

Abstract

A $\mathbb{Z}_4$-code of length n is a $\mathbb{Z}_4$-submodule of the $\mathbb{Z}_4$-module $\mathbb{Z}_4^n$. The dual code of a $\mathbb{Z}_4$-code is defined as its orthogonal complement with respect to the usual inner product on the module $\mathbb{Z}_4^n$. A $\mathbb{Z}_4$-code is self-dual if it is equal to its dual code. For $x \in \mathbb{Z}_4^n$, Euclidean weight of x is defined as $wt_E(x) = n_1(x) + 4n_2(x) + n_3(x)$, where $n_i(x)$ denotes the number of coordinates in x equal to i, for $i = 1, 2, 3$. A self-dual $\mathbb{Z}_4$-code can have codewords of Euclidean weight divisible by 4 or 8. If all codewords have Euclidean weight divisible by 8, then it is a Type II code. It is known that such codes can only exist for lengths divisible by 8. If the self-dual $\mathbb{Z}_4$-code is not a Type II code, it is a Type I code. The minimum Euclidean weight of Type II codes is at most $8\left\lfloor \frac{n}{24} \right\rfloor + 8$. The same bound holds for Type I codes of length $n \not\equiv 23 \pmod 4$. If minimum Euclidean weight of a self-dual $\mathbb{Z}_4$-code is equal to that bound, it is an extremal $\mathbb{Z}_4$-code. All self-dual $\mathbb{Z}_4$-codes are classified up to length 20. Also, extremal $\mathbb{Z}_4$-codes of length 24 are classified. Therefore, length 32 is the smallest length for which new extremal Type II codes can be found.

In this talk, we present some new extremal $\mathbb{Z}_4$-codes of lengths 32 and 40. To construct these codes we developed a search method based on random neighborhood search. With this method, we constructed at least 182 new Type II extremal codes of length 32 and at least 762 new Type I codes of the same length. Also, we constructed at least 40 new Type II $\mathbb{Z}_4$-codes and at least 4144 Type I extremal codes of length 40.

Ternary self-dual codes, Hadamard matrices and related designs

Vladimir D. Tonchev

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(Joint work with Sanja Rukavina, University of Rijeka, Croatia)

Abstract

Extremal ternary self-dual codes are known for the following lengths $n \equiv 0 \pmod{12}$: $n = 12$: the extended Golay code; $n = 24$: the extended quadratic-residue code [1] and the Pless symmetry code $C(11)$ [5], [6]; $n = 36$: the Pless symmetry code $C(17)$ [5], [6]; $n = 48$: the extended quadratic-residue code and the Pless symmetry code $C(23)$; $n = 60$: the extended quadratic-residue code, the Pless symmetry code $C(29)$, and the Nebe-Villar code [4]. According to the Assmus-Mattson theorem [1] every extremal ternary self-dual code of length divisible by 12 supports combinatorial 5-designs.

The Pless symmetry code $C(q)$ of length $n = 2q+2$, where $q \equiv -1 \pmod{3}$ is an odd prime power, contains a set of n codewords of weight n, which after replacing every entry equal to 2 with -1 form the rows of a Hadamard matrix equivalent to the Paley-Hadamard matrix of type II [6]. In particular, the Pless symmetry code $C(17)$ contains the rows of a Hadamard matrix P of Paley type II, having a full automorphism group of order $4 \cdot 17(17^2 - 1) = 19584$, and the rows of P span the code $C(17)$. It was shown in [8] that the code $C(17)$ contains a second equivalence class of Hadamard matrices equivalent to a regular Hadamard matrix H' such that the symmetric 2-$(36, 15, 6)$ design D has a trivial full automorphism group, and the row span of the incidence matrix of D over $GF(3)$ is a code equivalent to the Pless symmetry code $C(17)$.

Huffman [3] proved that any extremal ternary self-dual code of length 36 that admits an automorphism of prime order $p > 3$ is monomially equivalent to the Pless symmetry code. More recently, Eisenbarth and Nebe [2] extended Huffman's result by proving that the Pless symmetry code is the unique (up to monomial equivalence) ternary extremal self-dual code of length 36 that admits an automorphism of order 3. In addition, it was proved in [2, Theorem 5.1] that if C is an extremal ternary self-dual code of length 36 then either C is equivalent to the Pless symmetry code or the full automorphism group of C is a subgroup of the cyclic group of order 8.

In this talk, we report on the existence of a regular Hadamard matrix H^* which is monomially equivalent to the Paley-Hadamard matrix of type II such that the symmetric 2-$(36, 15, 6)$ design associated with H^* has a full automorphism group of order 24 and its (0,1)-incidence matrix spans a code equivalent to $C(17)$ [7]. Motivated by this and the results from [2], we classified all symmetric 2-$(36, 15, 6)$ designs that admit an automorphism of order 2 and their incidence matrices span an extremal ternary self-dual code of length 36 [7]. The results of this classification show that up to isomorphism, there exists exactly one symmetric 2-$(36, 15, 6)$ design D with an automorphism of order 2 that spans an extremal ternary self-dual code of length 36. The regular Hadamard matrix associated with D is equivalent to the Paley-Hadamard matrix of type II, and the ternary code spanned by the incidence matrix of D is equivalent to the Pless symmetry code.

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On two-weight codes invariant under the 3-fold covers of the Mathieu groups M_{22} and Aut(M_{22})

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Abstract

By a group representation theoretic approach, in [1] we construct quaternary $[693, 6, 480]_4$, $[1386, 6, 1008]_4$ and $[2016, 6, 1488]_4$ codes, and binary projective codes with parameters $[693, 12, 320]_2$, $[1386, 12, 672]_2$, $[2016, 12, 992]_2$ as examples of two-weight codes on which a finite almost quasisimple group of sporadic type acts transitively as permutation groups of automorphisms. In particular, we show that these codes are invariant under the 3-fold covers $\hat{3}M_{22}$ and $\hat{3}M_{22}{:}2$, respectively, of the Mathieu groups M_{22} and $M_{22}{:}2$. Using a known construction of strongly regular graphs from projective two-weight codes we obtain from the binary projective two-weight codes with parameters those given above, the strongly regular graphs with parameters $(4096, 693, 152, 110)$, $(4096, 1386, 482, 462)$, and $(4096, 2016, 992, 992)$, respectively. The latter graph can be viewed as a 2-$(4096, 2016, 992)$-symmetric design with the symmetric difference property whose residual and derived designs with respect to a block give rise to binary self-complementary codes meeting the Grey-Rankin bound with equality.

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Genetic algorithms in constructions of block designs and SRGs

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(Joint work with Dean Crnković)

Abstract

Construction of block designs with certain admissible parameters is often attempted for a particular set of parameters with the assumption of some additional constraints on the design structure in order to make the search computationally feasible. A natural constraint is the assumption that a given group of automorphisms acts on the design. One of the methods for constructing block designs with a prescribed automorphism group is the method that uses so called orbit matrices. It consists of two steps: construction of orbit matrices for the given automorphism group and construction of block designs for the orbit matrices obtained in this way (this step is called "indexing of orbit matrices"). Indexing is usually performed by exhaustive search. However, sometimes exhaustive search is not feasible because there are too many cases to check. Similarly, strongly regular graphs with certain admissible parameters can be constructed using orbit matrices for the prescribed automorphism group.

Genetic algorithms are search methods used in computing whose objective is to find exact or approximate solutions to optimization and search problems. A genetic algorithm mimics natural evolution, that is, it is based on optimizing a population (a subset of the entire search space).

In this talk, we will describe the use of a genetic algorithm in the step of indexing of orbit matrices for the construction of block designs and strongly regular graphs with a prescribed automorphism group.

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Codes from quasi-symmetric designs

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(Joint work with Dean Crnković, Doris Dumičić Danilović, Andrea Švob)

Abstract

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ be a t-(v, k, λ) design. For $0 \leq s < k$, s is called an intersection number of $\mathcal{D}$ if there exists $x, x' \in \mathcal{B}$ such that $|x \cap x'| = s$. A t-design is called quasi-symmetric if it has exactly two block intersection numbers x and y, $x < y$. In this talk, we give a construction of doubly even self-orthogonal codes from quasi-symmetric designs.

Block designs from self-dual codes obtained from Paley designs and Paley graphs

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(Joint work with Dean Crnković and Andrea Švob)

Abstract

In 2002, P. Gaborit introduced two constructions of self-dual codes using quadratic residues, so called pure and bordered construction, as a generalization of the Pless symmetry codes. In this talk, we give conditions under which the pure and the bordered construction using Paley designs and Paley graphs give self-dual codes. Special focus is on the binary and ternary codes. Further, we present t-designs from supports of the codewords of a particular weight in the binary and ternary codes obtained.

List of talks